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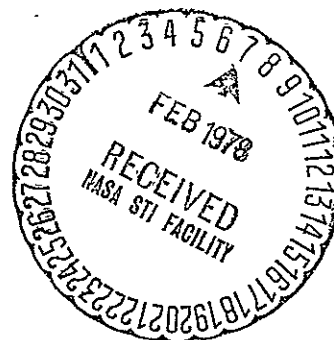
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# ELECTRICAL ENGINEERING AND COMPUTER SCIENCE



THE UNIVERSITY OF SANTA CLARA

FINAL TECHNICAL REPORT

NASA Grant NGR 05-017-031

Evaluation of STOL Navigation Avionics

December 1, 1977

## PREFACE

This final technical report covers work performed under NASA-Ames Grant NGR 05-017-031, Evaluation of STOL Navigation Avionics.

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III	Earth Electric Field Research - E-Field Model of Cumulonimbus Cloud
IV	Aircraft Noise Suppression - Electronic Acoustic Noise Suppression



## I. INTRODUCTION AND SUMMARY

The following summarize the research projects undertaken on NASA Grant NGR 05-017-031. Detailed research results are presented in Sections II through V.

*PROJECT:*           Application of Vector Magnetometer to Aircraft Control and Navigation.

*INVESTIGATORS:* W. R. Dunn and R. Pietila

Recent technological improvements and corresponding cost reductions in 3-axis (i.e., vector) magnetometry led to investigation by Ames and University of Santa Clara researchers into use of this device in avionics flight control and navigation applications. Early analytical work showed that vector magnetometer outputs could be used to determine aircraft change provided that one attitude angle was known. These results were confirmed by successful correlation of attitude computed from magnetometer data taken during a NASA Convair 880 test with actual aircraft attitude. To demonstrate the viability and uncover the limitations of the theory, development of a solid state remote magnetic indicator was undertaken. The effort involved development of a microprocessor based system employing a 3-axis magnetometer and evaluation of the system using test apparatus constructed at the University. The background and results of this work are described in detail in Section II of this report. Results of the research were also published in [1] and [2].

*PROJECT:*           Application of Earth Electric Field Phenomena to Aircraft Control and Navigation.

*INVESTIGATORS:* W. R. Dunn, C. Keller, W. Keller, R. Wilson, R. Yarbrough

Hill's [3] late 1972 paper on use of the earth's electric field in flight control prompted NASA and University researchers to undertake investigation of earth field applications to large aircraft control and navi-

gation.

The principle underlying Hill's work is the fact that in clear weather, the earth's surface terminates a large (approximately 300v/m at the surface) electric field which is close to the local vertical. Hill then successfully demonstrated that airborne (RPV) measurements of this field could be used to provide pitch/roll control.

University investigations were undertaken to evaluate the use of this phenomena in bad (e.g. IFR) weather conditions. The principal finding of this investigation was that this field (direction) can be significantly altered in and near cloud structures particularly the cumulonimbus cloud structure. Section III provides a review of the basic earth electric field phenomena and develops the basic field model for the cumulonimbus structure. The basic conclusion of this work is that the variability of the effects of the cumulonimbus on the clear weather electric field made navigational use of this field in all flight conditions a highly unreliable proposition. It is further proposed that electric field measurements might possibly better serve in a low cost system for identifying the direction and change intensity of cumulonimbus clouds during flight.

*PROJECT:*            Electronic Aircraft Cabin Noise Suppression

*INVESTIGATORS:* W. R. Dunn and W. Keller

Section IV describes an extension of Olson's [4] pioneering work in the use of active compensation as a means of reducing acoustic noise. This extended work provides a theoretical basis on which wide band acoustic noise can be significantly reduced in aircraft cabins.

*PROJECT:*            SIRU (Strapdown Inertial Reference Unit) Technical Support.

*INVESTIGATORS:* W. R. Dunn, W. Keller, C. Keller

University researchers worked with government and (government) contractor

personnel in the installation and checkout of the Carco motion simulator facility developed for use in the SIRU program. This effort consisted of Carco table orthogonality measurements, adjustment and calibration of recording electronics, and specification of Carco Table/Sigma 7 computer interface.

## Section II

### Aircraft Attitude Measurement Using a Vector Magnetometer

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## CHAPTER I

## A VECTOR AUTOPILOT SYSTEM

## 1-1 INTRODUCTION

An essential requirement of an aircraft attitude control system is that deviation of the body axes relative to a reference axes frame must be sensed. In addition, to overcome the ever-present possibility of errors or failure of the sensors, various configurations of redundant sensors are usually employed to assist in detection and correction of errors. To this end, there has been a continuing effort to improve existing sensors, to develop new sensor configurations, and to develop new sensor devices.

This chapter discusses the role of a vector magnetometer<sup>1</sup> as a new instrument for aircraft attitude determination. Although magnetometers have played a role in the attitude measurement of missiles and satellites [Ref. 1-1], there is an apparent lack of application in aircraft systems. By providing independent measures of attitude, the solid state vector magnetometer sensor system can not only assist in improving accuracy and reliability of existing systems but can also reduce component count with obvious benefits in weight and cost. Additionally, since a large number of aircraft heading reference systems depend on measurement of the Earth's magnetic field, it can be shown that by substituting a three-axis magnetometer for the remote sensing unit; both heading and attitude measurement functions can be derived using common elements, thereby further reducing the component count.

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<sup>1</sup>Aviation use to date has been essentially scalar magnetometry.

To investigate the feasibility of the above system, this chapter will proceed by developing a technique to determine attitude given magnetic field components. Sample calculations are then made using the Earth's magnetic field data acquired during actual flight conditions. Results of these calculations are compared graphically with measured attitude data acquired simultaneously with the magnetic data. The role and possible implementation of various reference angles are discussed along with other pertinent considerations. Finally, it is concluded that the Earth's magnetic field as measured by modern vector magnetometers can play a significant role in attitude control systems.

## 1-2 ATTITUDE DETERMINATION

Coordinate systems are usually defined by orthogonal right-handed sets of three unit vectors. An example of such a set is illustrated in Fig. 1-1 where the orientation of the body fixed frame used in this paper is delineated. Angular rotations are conventionally defined as rotations in the plane normal to a unit vector with the positive sense of rotation defined by the right-hand rule [Ref. 1-2].

To derive relationships of attitude variations as a function of magnetic vector component variation, we can proceed by considering matrix representations of an orthogonal transformation. If  $H_x$ ,  $H_y$ , and  $H_z$  are the magnetic components measured at a desired airframe attitude and  $H_x'$ ,  $H_y'$ , and  $H_z'$  are the components measured after any rotation of the body, vector  $H' = [H_x' \ H_y' \ H_z']^T$  can be related to vector  $H = [H_x \ H_y \ H_z]^T$  by an orthogonal linear transformation  $H' = AH$ . Here  $A$  must satisfy the orthogonality condition  $AA^T = I$ , where  $A^T$  is the transpose of  $A$ ; additionally, the determinant of  $A$  must be unity [Ref. 1-3, 1-4].



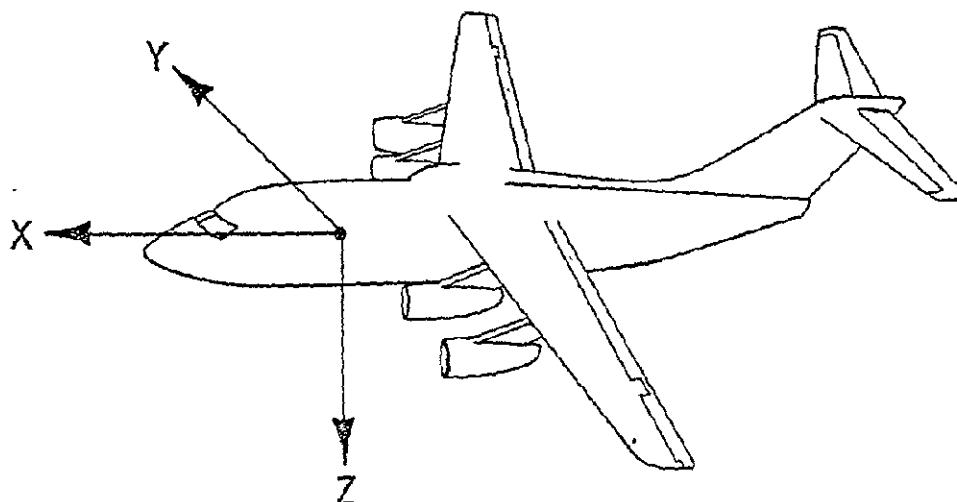


Fig. 1-1 AXIS ORIENTATION

Rotations about the z axis in Fig. 1-1 result in yaw deviations ( $\psi$ ) and in new components ( $H'$ ), as shown by

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-1)$$

Similarly, independent rotations about the y axis and the x axis result in pitch ( $\theta$ ) and roll ( $\phi$ ) dependent variations in the measured H components, as shown by

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-2)$$

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-3)$$

The effect of a combined rotation can be expressed by using the product of the transformation matrices. In addition, if the rotations are small, the total rotation experienced by applying sequential rotations is independent of the order in which the rotations are performed [Ref. 1-3,1-4].

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-4a)$$

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos \psi \cos \theta & \sin \psi \cos \phi + \sin \theta \cos \psi \sin \theta \\ -\sin \psi \cos \theta & \cos \psi \cos \phi - \sin \phi \sin \psi \sin \theta \\ \sin \theta & -\cos \theta \sin \phi \end{bmatrix}$$

$$\begin{bmatrix} \sin \phi \sin \psi - \sin \theta \cos \psi \cos \phi \\ \cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi \\ \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-4b)$$

Assume that the angular variations  $\theta$ ,  $\psi$ , and  $\phi$  are small enough so that the small angle approximations

$$\begin{aligned}\sin \theta &\approx \theta, \sin \psi \approx \psi, \sin \phi \approx \phi, \\ \cos \theta &\approx \cos \psi \approx \cos \phi \approx 1\end{aligned}$$

can be made. Then, if the products of small angles (in radians) can be assumed to be much smaller than the angles alone, the expression reduces to

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-5)$$

Further modifications in the form of the matrices result in

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} -Hz & Hy & 0 \\ 0 & -Hx & Hz \\ Hx & 0 & -Hy \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} + \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \quad (1-6)$$

By subtracting, we arrive at an expression for the difference in H components as functions of angular deviation.

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} - \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} = \begin{bmatrix} \Delta Hx \\ \Delta Hy \\ \Delta Hz \end{bmatrix} = \begin{bmatrix} -Hz & Hy & 0 \\ 0 & -Hx & Hz \\ Hx & 0 & -Hy \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} \quad (1-7)$$

It is significant to note at this point that the transformation matrix is singular implying that solutions for  $\theta$ ,  $\psi$ , and  $\phi$  are not independently available.

### 1-3 ATTITUDE DETERMINATION EMPLOYING MAGNETIC FIELD COMPONENTS

A given orthogonal set of three unit vectors can be displaced in Euclidean space by rotating the system through any angle  $\delta$  about a directed rotation axis. It is also customary to represent this rotation vectorially as a directed line segment whose length is proportional to the rotation angle. This rotation is analogous to the rotation experienced by the body fixed frame of Fig. 1-1 as the aircraft experiences combined pitch, yaw, and roll variation. During flight the body fixed set rotates about this rotation axis assuming new (possibly erroneous) attitudes in space. The task of the attitude sensing system is to provide measures of compounded pitch, yaw, and roll that would result in the same attitude assuming that the rotations occurred sequentially about the x, y and z axes rather than the actual rotation axis.

It was shown in the previous section that a compounded rotation of an orthogonal set can be described by a product of respective transformation matrices. Additionally it was noted that for small angular rotations the order of multiplication is unimportant. Using the relationships of (1-7), expressions for the angular deviations in terms of measured magnetic vector components can be derived.

$$\Delta H_x = -H_z\theta + H_y\psi \quad (1-8a)$$

yields

$$\theta = (H_y\psi - \Delta H_x)/H_z \quad (1-8b)$$

$$\psi = (\Delta H_x + H_z\theta)/H_y \quad (1-8c)$$

Similarly,

$$\Delta H_y = -H_x \psi + H_z \phi \quad (1-9a)$$

yields

$$\psi = (H_z \phi - \Delta H_y) / H_x \quad (1-9b)$$

$$\theta = (\Delta H_y + H_x \psi) / H_z \quad (1-9c)$$

and

$$\Delta H_z = H_x \theta - H_y \phi \quad (1-10a)$$

yields

$$\theta = (\Delta H_z + H_y \phi) / H_x \quad (1-10b)$$

$$\phi = (H_x \theta - \Delta H_z) / H_y \quad (1-10c)$$

Assuming that  $H_x$ ,  $H_y$  and  $H_z$  are nominal vector components as measured in a reference attitude and that  $H_x'$ ,  $H_y'$  and  $H_z'$  are new field components at the new attitude, then  $\Delta H_x = H_x' - H_x$ ,  $\Delta H_y = H_y' - H_y$ ,  $\Delta H_z = H_z' - H_z$  are expressions of the incremental changes in field components. Additionally, before using (1-8), (1-9) or (1-10) to solve for attitude variations (pitch, yaw, or roll), one additional angle from an auxiliary sensor<sup>2</sup> must be supplied. Using one additional angle of rotation (about any one axis) the remaining two rotations can then be calculated.

To illustrate this point, flight data acquired during the flight of a NASA flown Convair 900 instrumented with a three-axis magnetometer and a Litton inertial navigation system were used to calculate roll, pitch, and yaw.

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<sup>2</sup>It was noted following (1-7) that a unique solution for attitude variation is not possible using magnetic field data alone.

Attitude variation about each of the three axes was calculated using measured magnetic field components supported by one angle from the inertial system. The results of these calculations are plotted in Figs. 1-2 through 1-4.

It is significant to note that the rotations shown occurred simultaneously (i.e., time base is the same for all three figures). The flight was at an altitude of approximately 5000 ft at an airspeed of approximately 250 nmi/h.

Although the data used to plot the attitudes shown in Figs. 1-2 through 1-4 were not acquired specifically for this purpose, the correlations in measured and calculated attitude clearly show that, within the limits of instrument accuracy, signals proportional to attitude variation can be derived using flight data.

#### 1-4 A POSSIBLE SYSTEM CONFIGURATION

Since the intent of this chapter is to introduce the notion that magnetometer technology has advanced to the point where three-axis magnetometers can be incorporated in aircraft attitude sensing systems on a cost effective basis, the system discussion will be limited in scope to describing a possible combined heading and attitude measurement method.

Heading references fall into three classes; 1) those that depend on the Earth's magnetic field, 2) those that depend on the use of low-drift gyroscope to retain a preset azimuth, and 3) those (gyrocompasses) that depend on sensing the Earth's rotation [Ref.1-5]. By far the greatest number of aircraft heading systems depend on the Earth's magnetic field, although many of these include gyroscopes to improve the performance characteristics.

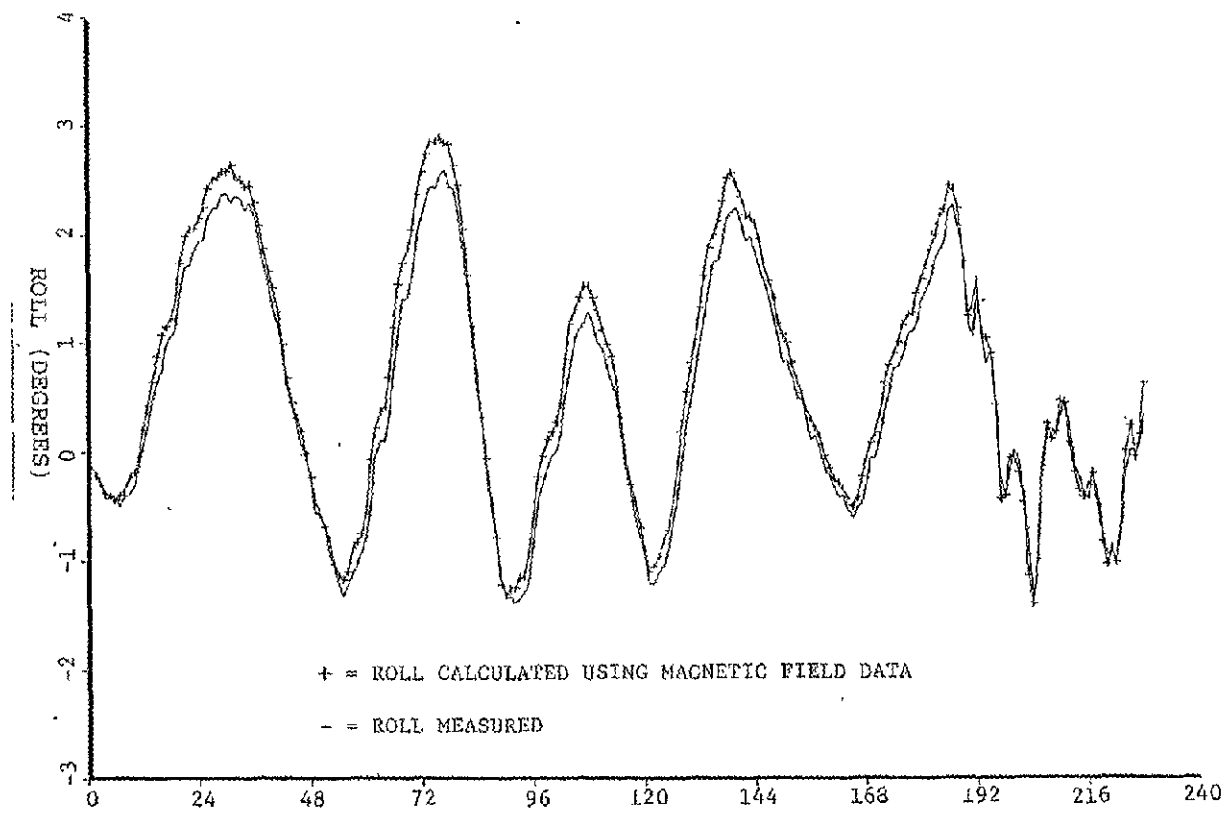


FIGURE 1-2 ROLL AXYS

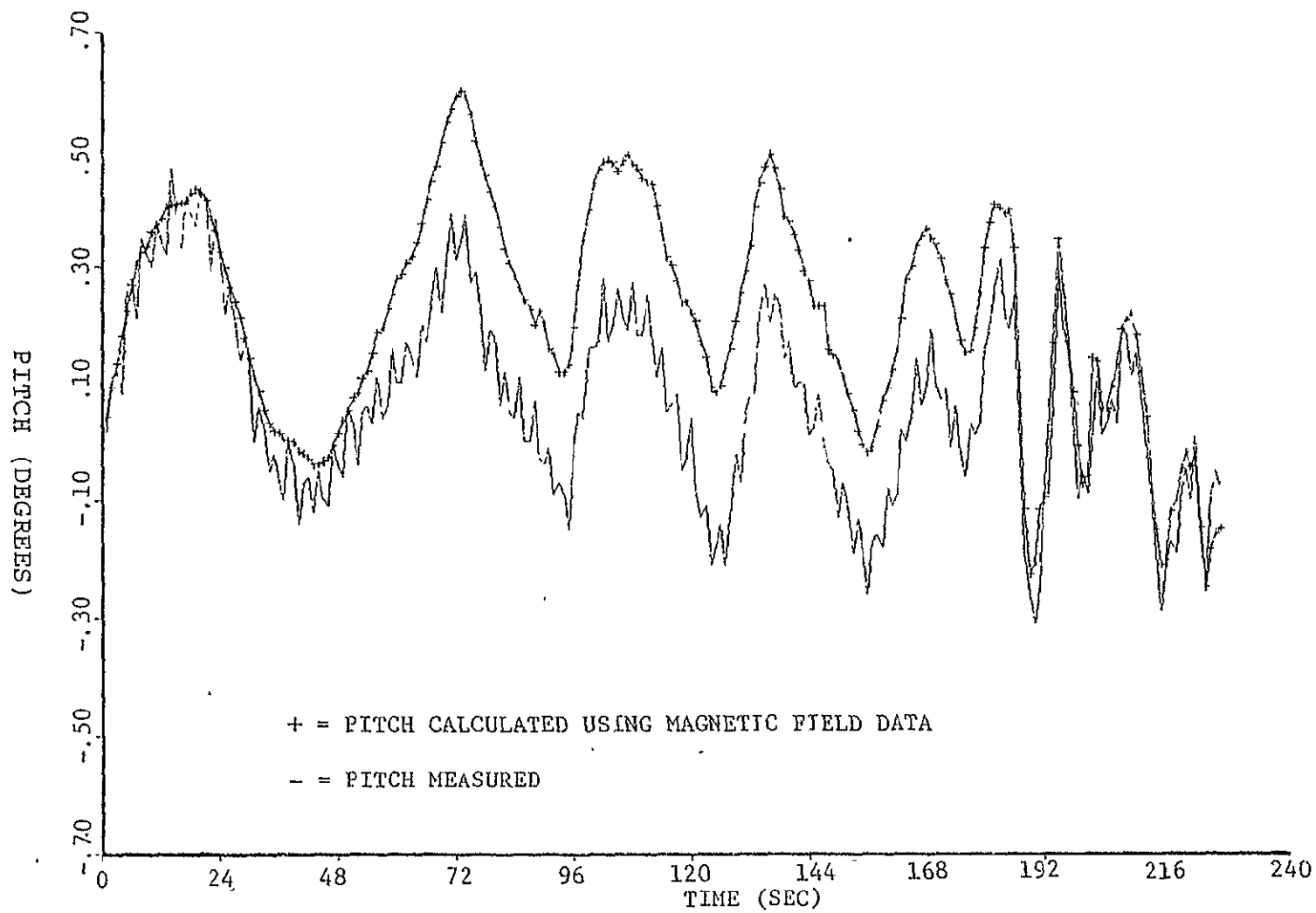


FIGURE 1-3 PITCH AXIS



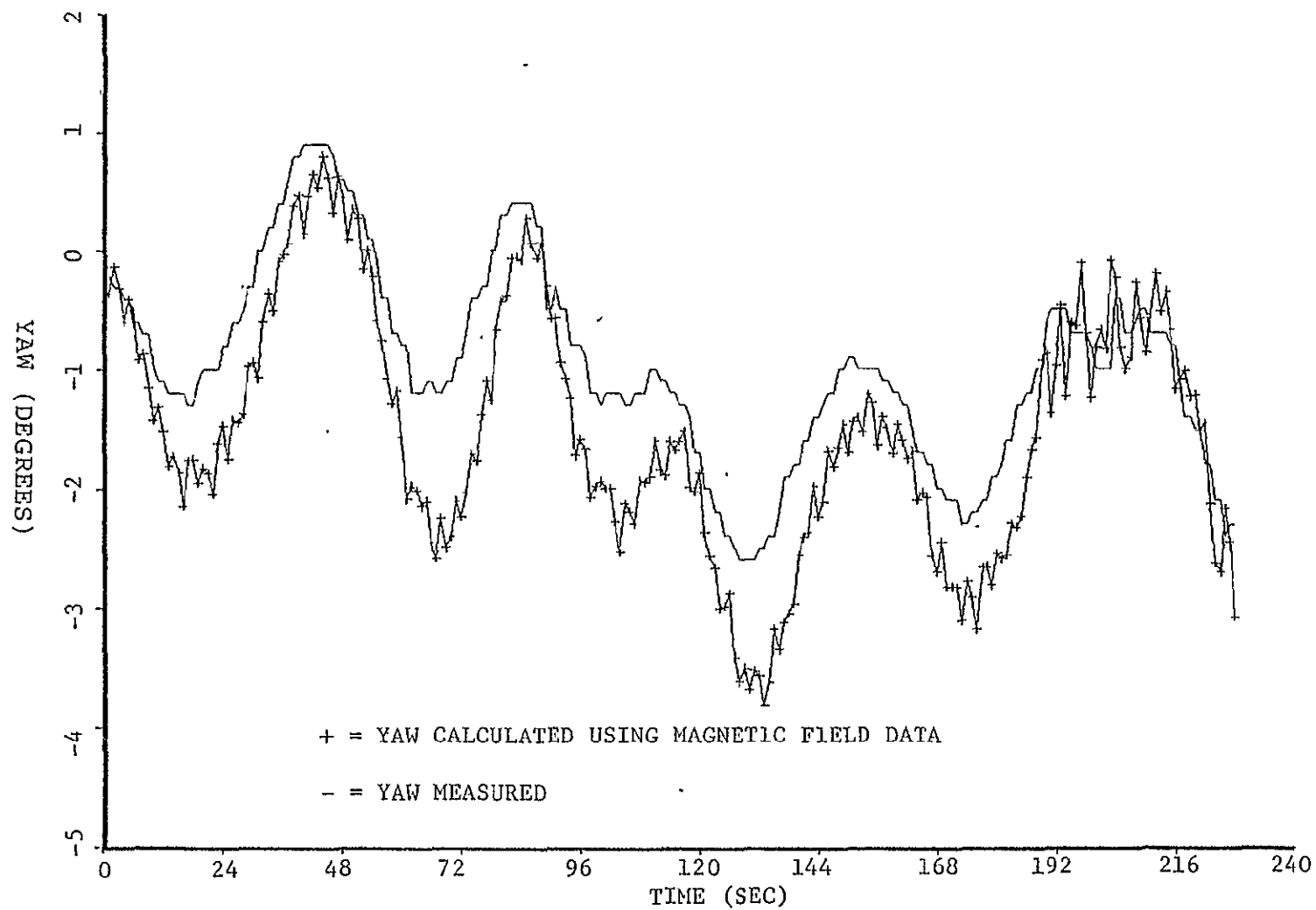


FIGURE 1-4 YAW AXIS

A popular system combination (with no gyro) is to combine a pendulous remote magnetic sensor and a synchro receiver in a null seeking circuit. The philosophy being to attempt to measure only the horizontal component of the Earth's magnetic field and to swing the receiver into alignment with it. Under acceleration, departures of the sensor unit from the horizontal result in angular heading errors  $\epsilon$  [Ref. 1-5].

$$\epsilon = (aH/g) \tan \gamma \sin \theta$$

where  $aH$  is the horizontal acceleration,  $g$  is the acceleration due to gravity,  $\theta$  is the angle between the acceleration vector and magnetic north, and  $\gamma$  is the magnetic field dip angle;  $\arctan$  (vertical field/horizontal field).

Accuracy of this system can be improved by incorporating a strapped-down solid state magnetic sensing unit (free of acceleration errors) that measures and displays the angle of the Earth's horizontal magnetic component relative to the aircraft. This system can be implemented as follows:

1) Determine the direction of the magnetic vector  $F$  relative to the sensors (and the airframe), by measuring the  $x$ ,  $y$  and  $z$  components (Figs. 1-1 and 1-5). The direction cosines  $\cos \alpha$ ,  $\cos \beta$ ,  $\cos \gamma$  are the cosines of the angles  $\alpha$ ,  $\beta$ ,  $\gamma$  between the magnetic vector and the positive  $x$ ,  $y$  and  $z$  axes. Additionally,

$$\cos \alpha = x / (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\cos \beta = y / (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\cos \gamma = z / (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

- 2) Using either a vertical reference<sup>3</sup> or knowledge of aircraft attitude, we can effectively rotate the body axes such that the x-y plane is horizontal (see Chapter II).
- 3) Simple application of direction cosines will yield the direction of magnetic north in the aircraft's x-y plane.

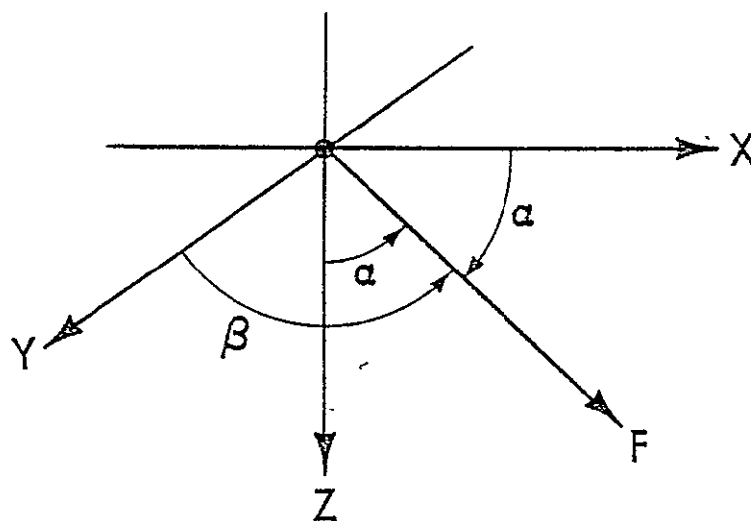


Fig. 1-5 FIELD VECTORS AND DIRECTION COSINES

Although the preceding discussion implies that heading can be determined by using a strapped-down magnetometer, there remains the problem of attitude determination. Another widely used system for obtaining a heading reference

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<sup>3</sup>Not necessarily derived inertially [Ref. 1-11].

is to combine the relatively excellent short term stability of a directional gyroscope with the long term stability of magnetic field measurements. By slaving the directional gyroscope to the magnetic field [Ref. 1-5, sec. 10.4.7], gyroscopes with relatively large free drift error can be used to provide an excellent heading reference.

Replacement of the pendulous remote sensing unit of this type of system with a strapped-down vector magnetometer would result in both heading and attitude information on a continuous basis. This combination would operate as follows:

- 1) The system is initialized by determining a reference attitude (perhaps by using a primary inertial attitude system).
- 2) The angular position of the horizontal magnetic field component is computed as above and used to slave the directional gyroscope.
- 3) The directional gyroscope, with relatively good short term stability (devices with free drift of less than 0.5 deg/h have been designed), is used to determine yaw ( $\psi$ ) errors.
- 4) For small angle deviations, (1-8), and (1-9), and (1-10) can be employed to recalculate aircraft attitude. The process loops back to step 2) closing the loop on a combined attitude and heading reference system.

The sampling frequency required to maintain an acceptable level of error is of course determined by the aircraft performance expected (angular rates) and by the gyro error (drift rate plus errors due to additional sources such as

gyroscope tilt from vertical). The overall system is such that heading can be determined as before with errors due to sensor departures from horizontal substituted for long term accumulation of attitude uncertainty (this can be corrected by looping to step 1) at a frequency dependent on error rates). Additionally one gains measurements of attitude with minimal computation and replacement of a mechanical remote sensing unit with a solid state strapped-down magnetometer sensor.

#### 1-5 OTHER CONSIDERATIONS

The characteristics of the Earth's magnetic field and its variations have long been established [Ref. 1-6-1-10]. Since the field is to be used as a reference in the attitude measurement scheme, there is a need here to discuss its adverse characteristics. Although the field does experience variation, most of the variation is either in amplitude (ionospheric contributions) or has time constants that make the variation negligible (secular variation).

In traversing local anomalies, there will, however, be deflections in the ambient field due to the additive effect of local dipoles or monopoles. The effect of local terrain caused anomalies can be visualized by picturing the main field vector oriented in space with a second modulating vector rotating at its tip. Maximum angular error would occur when this modulating vector has maximum magnitude and is positioned at right angles to the main vector.

To illustrate the effect of local anomalies one can calculate the level of anomaly required to cause an error. Since the Earth's main field is typically in the order of 0.50 G it is readily apparent that a local anomaly of approximately 0.01 G at right angles to the local field is

required to cause an error of 1 deg. Furthermore, the local anomaly would have to be aligned with one of the aircraft body axes to result in one degree of attitude error in any one axis. Fortunately, anomalies with components of this magnitude positioned at right angles to the main field are extremely rare. In addition, the anomalies are localized over ore bodies or other geophysical irregularities, have magnitudes that diminish as the cube of altitude, and tend to average to zero over relatively short distances. In summary, the probability of encountering an anomaly that would cause as much as a 1 degree error is relatively small. The error, if introduced, will be short lived and, unlike drift error, will average to zero.

Fundamental to a magnetic field referenced system is the ability to measure orthogonal components of the field vector. Precision and accuracy of measurement of the components is of course specified by the desired control specifications.

Since the Earth's magnetic field varies in magnitude on a global basis between 0.3 G and 0.6 G (30,000 gamma to 60,000 gamma), it is apparent that full scale measurements of 0.6 G can be expected. Sensors mounted at right angles to the field will monitor no measureable field and thus define the lower limit of measurement to be zero. For the continental United States the declination varies between 60 and 80 deg, resulting in a range in horizontal component of 0.15 to 0.25 G with vertical component in the range of 0.4 to 0.55 G. Heading variations (yaw) result in changes of the horizontally sensed field components and would specify the maximum precision required. In addition, flight at  $45 \text{ deg} \pm (n \times 90 \text{ deg})$  (where n is any whole number) with respect to magnetic north results in minimum sensitivity of the x and y

axes measurements. In this case sensor inputs would range between 0.106 and 0.177 G with minimum field at the north. Assuming the preceding ambient measurements, variations in component magnitude of approximately 0.0180 to 0.0305 G/deg for small angle variations can be expected.

A brief survey of commercial magnetometer manufacturers reveals that triaxial magnetometers that measure from zero to 0.6 G with linearities of 0.5 percent, noise less than  $\pm 1$  mG and sensitivities of at least 2.5 V per 600 mG are currently available. In addition, these devices have a bandwidth of direct current to at least 500 Hz and are rated to have less than 1 deg error in orthogonality.

From a precision standpoint, it is apparent that variations in yaw for this worst case situation can be sensed to better than 0.1 deg with currently available magnetometer technology. The sensor technology required to implement an attitude sensing system of reasonable specifications is available (more detailed analysis is presented in Chapter III).

Although the preceding calculations indicate that for small angular variations attitude can be calculated using measured magnetic data, there is a need to consider the effects of larger finite rotations. In this case the small angle assumptions would not be valid and an Euler transformation would have to be made. Measurement of three axes of field components could be used to develop the direction cosines required to determine the orientation of the axis of rotation, the angular rotation about it, and the three angular rotations of pitch, roll, and yaw.

For the special case where the axis of rotation aligns with the magnetic vector, there would of course be no

measured component changes.<sup>4</sup> By measuring the attitude of a second vector (not in alignment with the magnetic vector), we could resolve the ambiguous situation cited above and provide additional redundancy.

The optimum auxiliary vector would be one that could be sensed without using inertial devices. The Earth's electric field can be considered. The main reason for considering this field as a means of providing an auxiliary angular reference is that the resultant system has the potential of being completely solid state. The electric field vector can be used to determine attitude variation in a manner analogous to the magnetic vector system. Inherent limitations of each single vector system can be obviated if the vectors are not coincident.

Although Hill [Ref. 1-11] reported success in controlling pitch and roll using the electrostatic field alone, comments by Markson [Ref. 1-12] indicate that the electrostatic field is not always a reliable vertical reference. Employment of the electrostatic field for this attitude measurement system is limited to augmenting the magnetic field measurements by eliminating ambiguity of motion around the magnetic vector. The requirement of vertical electrostatic field is thus removed and replaced by a requirement that the field direction is relatively stable.

By using two independently derived vectors we have sufficient data to obviate the ambiguity just cited and we have the potential of providing redundancy as well.

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<sup>4</sup>An example of this would be yaw rotation while flying straight and level over the magnetic poles or roll rotation while flying towards a pole at the magnetic equator.



## 1-6 CONCLUSION

This chapter has identified a novel method of measuring aircraft attitude using relatively inexpensive, well developed instrumentation. It has recognized that magnetic field sensing systems have been used to some extent in attitude sensing and control of space vehicles; it has also suggested, however, that with appropriate support, magnetometers can find increased application in aircraft attitude measurement systems.

This claim is corroborated by actual flight test data. Magnetometers have evolved to a point where three axis measurements of the Earth's magnetic field can be made with sufficient precision and accuracy to enable measurement of small angle attitude variations.

This chapter has also discussed a possible system configuration combining heading determination and attitude measurement functions. By replacing the conventional remote sensing unit with a three-axis magnetometer, it has been suggested that both functions can be obtained with the hardware required previously for heading measurement alone.

As with any system, there are limitations imposed. The main limitation for a vector magnetometer system seems to be the inability to sense rotations around the magnetic vector itself. This problem is not unlike the ambiguity experienced by magnetic heading systems at high latitudes. By judiciously incorporating auxiliary instruments, not only can the ambiguities be removed but a degree of redundancy can be added while still maintaining a cost and weight advantage over comparable systems.

## CHAPTER II

### AN ATTITUDE INDEPENDENT REMOTE MAGNETIC INDICATOR

#### 2-1 INTRODUCTION

Preliminary investigation [Ref.2-1] revealed that aircraft attitude can be calculated using measurements of earth's magnetic field vector and a single auxiliary rotation angle. An algorithm to compute the two remaining aircraft rotational angles was developed. Using flight data, it was demonstrated that an excellent correlation in computed versus actual aircraft attitude could be achieved. In addition to providing measurements of the magnetic field for redundant attitude computations (to improve accuracy and reliability of existing autopilot systems), it was noted that the vector magnetometer could substitute for the remote magnetic sensing unit. In this manner both heading and attitude measurements could be derived using common elements with obvious benefits in weight and cost.

This chapter discusses the mechanization of a microprocessor based computer system that uses a three axis magnetometer plus gyro data to compute heading. The magnetometer is a three axis solid state device that can be mounted in a strapped down configuration resulting in an attitude independent remote magnetic indicator. Gyro measurements of pitch and roll angle plus three axis magnetic measurements are used by the algorithm to compute aircraft heading. The system can function independently to compute heading or by simply increasing the stored program could implement the attitude computing algorithm of [Ref. 2-1] as well.

The chapter proceeds by developing an algorithm to compute aircraft heading using the strapped down magnetometer

and two gyro measured angles. Practical aspects of designing the system including both hardware and software are then presented. In addition, the limitations in instrument accuracy and operation as determined by sensor errors, signal processing errors, arithmetic precision and computation speed are discussed. Considerable computational capability inherent in the system enables minimization of systematic errors. It is demonstrated that inexpensive sensors can be employed with offset and orthogonality errors compensated by microprocessor programming. Finally, it is concluded that a microprocessor based computer with a solid state magnetometer can play a significant role in aircraft instrumentation.

## 2-2 AN ALGORITHM TO COMPUTE AIRCRAFT HEADING

Coordinate frames are usually defined by orthogonal right-hand sets of three unit vectors. An example of such a set is illustrated in Fig. 1-1 where the orientation of the body fixed frame used in this chapter is delineated. The reference coordinate frame referred to in this chapter is oriented with axes  $x$  and  $y$  in the horizontal plane and axis  $z$  vertical ( $z$  down is positive). Pitch attitude angle ( $\theta$ ) of an aircraft is defined [Ref. 2-2] as the angle between some preferred longitudinal axis and the horizontal reference. In this chapter, pitch angle is the angle between the  $x$  axis of the aircraft and the  $x$ - $y$  plane of the reference axis set. Since angular rotations are conventionally defined as rotations in the plane normal to a unit vector with the positive sense of rotation defined by the right-hand rule [Ref. 1-2], we will define positive pitch angle ( $\theta$ ) as the "nose up" or positive rotation about the  $y$  axis when the  $y$  axis is horizontal. The roll and yaw angles ( $\phi$  and  $\psi$ ) will then simply be rotations about the  $x$  and  $y$  axes respectively.

By aligning the three magnetometer axes with the respective x, y and z axes of the aircraft, we can measure magnetic field components of the aircraft at any attitude. For the trivial case where pitch ( $\theta$ ) and roll ( $\phi$ ) are both zero degrees, Hx and Hy are the horizontal field components and we can compute yaw from the horizontal vectors as follows:

$$\psi_1 = \cos^{-1}(H_x / (H_x^2 + H_y^2)^{\frac{1}{2}}) \quad (2-1a)$$

or

$$\psi_1 = \sin^{-1}(H_y / (H_x^2 + H_y^2)^{\frac{1}{2}}) \quad (2-1b)$$

We select either (2-1a) or (2-1b) based on the relative magnitudes of Hx and Hy. By minimizing the numerator of the argument we guarantee that the inverse trigonometric operation results in an angle between zero and forty-five degrees with maximum sensitivity ensured. Heading is then computed using the signs of Hx and Hy to select the appropriate equation from Table 2-1.

<div style="text-align: center;"> <div style="display: inline-block; transform: rotate(-45deg);"> Hx Hy </div> </div>	Hx	
	NEGATIVE	POSITIVE
Negative	$\psi = 180 - \psi_1$	$\psi = \psi_1$
Positive	$\psi = \psi_1 + 180$	$\psi = 360 - \psi_1$

Table 2-1. Formulae to Compute Heading

For most cases, the pitch and roll angles are not zero and inverse rotations are required to determine the actual horizontal field components Hx and Hy. Since any aircraft attitude can be represented as a sequence of rotations about each axis beginning at some reference attitude, we can

determine the reference Hx and Hy field components by performing an inverse roll followed by an inverse pitch computation<sup>1</sup>.

The inverse roll computation can be developed by considering vector components of an arbitrary vector  $\bar{H}$  in Fig. 2-1. The first set ( $x_2, y_2, z_2$ ) represents the vector components measured in a reference orientation. The second set has common origin and aligns with common x axis component. It is rotated (rolled) about the x axis resulting in new y and z values. We can describe vector  $\bar{H}$  in both coordinate frames as

$$\bar{H} = x_2 \cdot \hat{i}_2 + y_2 \cdot \hat{j}_2 + z_2 \cdot \hat{k}_2 \quad (2-2)$$

and

$$\bar{H} = x_3 \cdot \hat{i}_3 + y_3 \cdot \hat{j}_3 + z_3 \cdot \hat{k}_3 \quad (2-3)$$

Since the vector  $\bar{H}$  is unique, we note that equations (2-2) and (2-3) are equal. Furthermore if we form dot products we solve for the horizontal components  $x_2, y_2$ , and  $z_2$  in terms of the rotated values and the roll angle ( $\phi$ ).

From (2-2) we obtain

$$\bar{H} \cdot \hat{i}_2 = x_2 (\hat{i}_2 \cdot \hat{i}_2) + y_2 (\hat{j}_2 \cdot \hat{i}_2) + z_2 (\hat{k}_2 \cdot \hat{i}_2) \quad (2-4a)$$

$$\bar{H} \cdot \hat{i}_2 = x_2 \quad (2-4b)$$

and from (2-3) we obtain

$$\bar{H} \cdot \hat{i}_2 = x_3 (\hat{i}_3 \cdot \hat{i}_2) + y_3 (\hat{j}_3 \cdot \hat{i}_2) + z_3 (\hat{k}_3 \cdot \hat{i}_2) \quad (2-5a)$$

$$\bar{H} \cdot \hat{i}_2 = x_3 \quad (2-5b)$$

---

<sup>1</sup>Since pitch is defined as the angle between the x axis and the horizontal plane we can assume that at any heading, aircraft attitude results due to a pitch followed by a roll.

then

$$x_2 = x_3 \quad (2-6)$$

Similarly,

$$\bar{H} \cdot \hat{j}_2 = y_2 = x_3 (\hat{i}_3 \cdot \hat{j}_2) + y_3 (\hat{j}_3 \cdot \hat{j}_2) + z_3 (\hat{k}_3 \cdot \hat{j}_2) \quad (2-7a)$$

$$y_2 = y_3 \cos \phi - z_3 \sin \phi \quad (2-7b)$$

and

$$\bar{H} \cdot \hat{k}_2 = z_2 = x_3 (\hat{i}_3 \cdot \hat{k}_2) + y_3 (\hat{j}_3 \cdot \hat{k}_2) + z_3 (\hat{k}_3 \cdot \hat{k}_2) \quad (2-8a)$$

$$z_2 = y_3 \sin \phi + z_3 \cos \phi \quad (2-8b)$$

These expressions can be summarized as

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \quad (2-9)$$

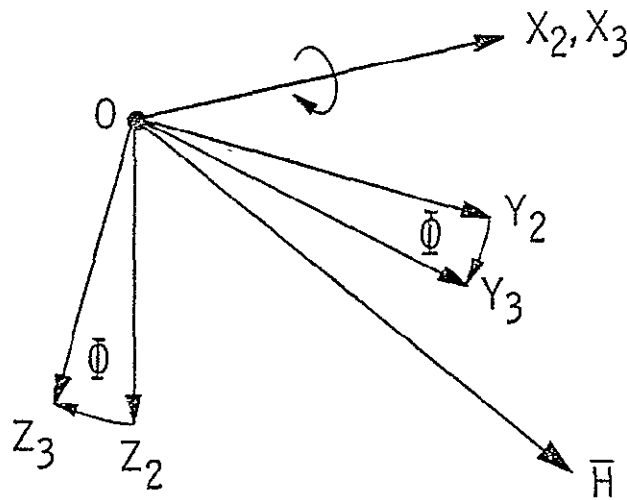


Fig. 2-1 AXES ROTATED IN ROLL

Similarly, considering an axis set rotated in pitch as shown in Fig. 2-2, we can express the reference set  $x_1, y_1, z_1$  in terms of the rotated set  $x_2, y_2, z_2$  as follows

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \quad (2-10)$$

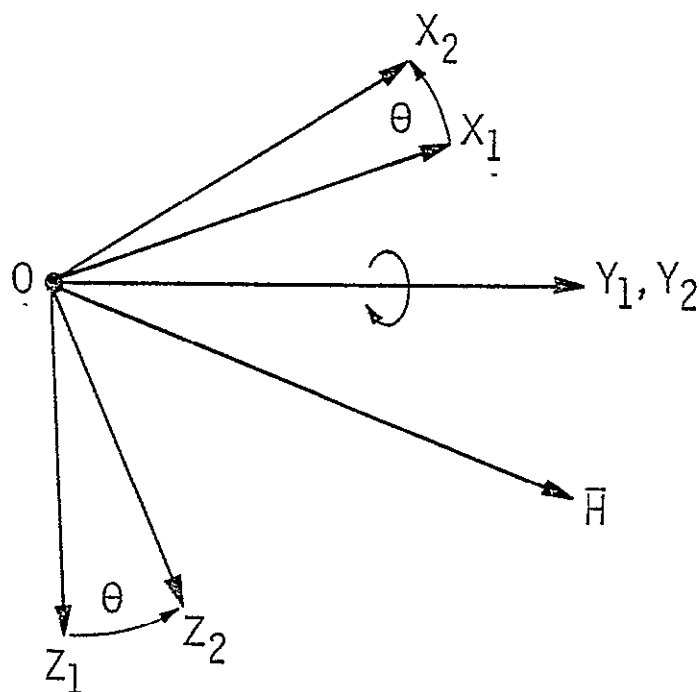


Fig. 2-2 AXES ROTATED IN PITCH

Finally, if we assume that the axis set subscripted with <sup>3</sup> represents components of Earth's magnetic vector measured at an arbitrary aircraft attitude, we can derive the magnetic components ( $H_{xh}, H_{yh}, H_{zh}$ ) in the horizontal plane for a given heading

$$\begin{bmatrix} H_{xh} \\ H_{yh} \\ H_{zh} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \cdot \begin{bmatrix} H_{x_3} \\ H_{y_3} \\ H_{z_3} \end{bmatrix} \quad (2-11a)$$

$$\begin{bmatrix} H_{xh} \\ H_{yh} \\ H_{zh} \end{bmatrix} = \begin{bmatrix} \cos \theta & (\sin \theta \sin \phi) & \sin \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ -\sin \theta & (\cos \theta \sin \phi) & \cos \phi \cos \theta \end{bmatrix} \cdot \begin{bmatrix} H_{x_3} \\ H_{y_3} \\ H_{z_3} \end{bmatrix} \quad (2-11b)$$

The algorithm to be implemented with the microprocessor would therefore require operations as outlined in Fig. 2-3. Details of programming method, modifications to the above equations to facilitate programming and computation speed versus accuracy tradeoffs are discussed in following sections.

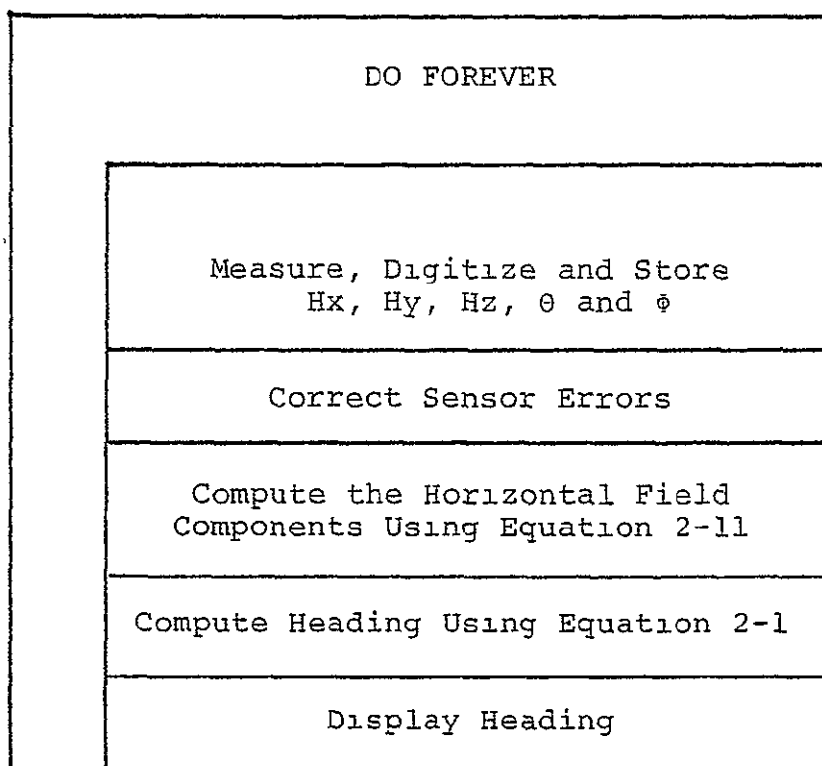


Fig. 2-3 LOGICAL OPERATIONS REQUIRED TO COMPUTE HEADING



## 2-3 MECHANIZATION OF THE HEADING ALGORITHM

### A. General Considerations

To evaluate the performance of an integrated system experimentally, an instrument was designed to implement the algorithm developed above. Several approaches were considered to implement the heading instrument for experimentation:

- 1) A minicomputer implementation incorporating an HP-2100 minicomputer supported by peripheral interface and analog circuitry. Programming of the HP-2100 would have enabled the computer to control multiplexing and processing of sensor data as suggested by Parish and Lee [Ref. 2-3].
- 2) A hybrid system composed of a remote data acquisition system to collect data from sensors for subsequent processing by a computer (possibly an HP-2100).
- 3) A digital/analog electronic implementation incorporating the design of a special purpose computer to perform the required functions of a heading instrument.

The first two approaches were abandoned since it was desirable to perform the experiments at various locations remote from a computer facility and to have data available immediately without having to rely on off-line computations at a later date. The design task then evolved to the design of a special purpose computer system to implement the algorithm, provide a means for evaluating the performance of the proposed algorithm and to allow modifications to the system if required.

### B. Design Criteria

Having decided on the general approach to implementing

the algorithm it became necessary to consider the performance criteria desired of the instrument.

1) Accuracy

As a design goal, an absolute accuracy of  $\pm 1.0^\circ$  in heading uncertainty was selected for the laboratory implementation. This accuracy is compatible with commercially available heading systems.

2) Computation Speed

The bandwidth of the system is determined mainly by the computation speed of the computer<sup>2</sup>. As a design goal, complete heading updates once per second was established.

3) Flexibility

A desirable feature of the laboratory evaluation instrument was considered to be flexibility. Revisions or additions to the algorithm as predicted by experimental data should be incorporated with minimal redesign of the instrument.

## 2-4 CONCLUSIONS

An instrument designed to implement the heading algorithm developed above uses a three axis magnetometer to measure magnetic field data in the vicinity of an aircraft. Since the magnetometer proposed is a solid state three axis fluxgate device and is permanently mounted in a strapped down configuration, the implementation results in an attitude independent

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<sup>2</sup>The response times of the various sensors and analog circuitry are orders of magnitude greater than the desired one second sample interval.

remote magnetic indicator<sup>3</sup>.

Several factors will contribute to system inaccuracy. Although the major error sources can be evaluated mathematically (Chapter IV), there is a need to evaluate the implementation experimentally. Systematic errors that arise can be reduced by instrument computation. This capability (inherent with a computer based system) enables incorporation of less expensive sensors in the heading instrument with less concern with factors such as temperature regulation, sensor orthogonality and sensor offset<sup>4</sup>.

Since the algorithm can be implemented using a microprocessor as the major computer element, the resulting instrument will have inherent computation capability, be small in size and consume relatively little power. These factors make the instrument an ideal device for aircraft application where the need for redundant distributed processing capability is invaluable.

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<sup>3</sup>Current remote magnetic indicators are pendulous and rely on gravity to enable measurements of the horizontal magnetic vector (not attitude independent).

<sup>4</sup>Assuming that the sensors have repeatable or measurable characteristics, algorithms can be developed to correct previously measured erroneous data.

## CHAPTER III

### DESIGN OF A MICROPROCESSOR BASED HEADING INSTRUMENT

#### 3-1 INTRODUCTION

Progress in device and component technologies during the 1970's has led to an assortment of sophisticated integrated circuits (IC) devices [Ref. 3-1] which enable the design of instruments with a high degree of sophistication and accuracy. Of these devices, the microprocessor has to date been the most exploited component in industrial control and instrumentation applications [Ref. 3-2 through 3-7]. There have been many papers presented addressing the general application and feasibility of applying microcomputers to particular design tasks [Ref. 3-8 through 3-21].

Although much of the literature to date on microprocessors has addressed the design of commercial products (usually the final result of a carefully orchestrated effort beginning with a market survey), the design of a laboratory instrument for algorithm evaluation differs in design philosophy. In particular, the laboratory instrument is designed to evaluate a proposed algorithm under laboratory conditions. The traditional benchmark evaluations and attempts to match the microprocessor to the application is not only difficult but unnecessary. If the processor is much more powerful than necessary, the "overkill" is little noticed; but if an insufficiently endowed microprocessor is selected, the effects can be devastating. Not only will the program be difficult to write and voracious of memory, it would be difficult to change to a more powerful microprocessor part way through the project. With these considerations in mind, a general purpose, flexible microprocessor with powerful architecture and instruction set the Signetics 2650 microprocessor [Ref. 3-22] was selected.

### 3-2 HARDWARE DESIGN CONSIDERATIONS

The design of a microprocessor based system begins by considering the total system level block diagram to be implemented (Fig. 3-1). Inputs from five sensors including x, y and z axis magnetic data plus pitch and roll angles ( $H_x$ ,  $H_y$ ,  $H_z$ ,  $\theta$  and  $\phi$ ) are to be multiplexed, sequentially sampled and converted to a digital representation prior to processing (executing the algorithm developed above). The main subsystem of Fig. 3-1, the central processing unit (CPU), operates under control of instructions stored in the system memory and interfaces with the input and output subsystems via data ports.

At this early stage in the design, it is significant to note that the block diagram of Fig. 3-1 differs slightly from that of a classical discrete hardware solution. The input subsystem (composed of analog multiplexer, sample and hold, and analog to digital converter) differs from a conventional data acquisition in that it is devoid of a control section. The microprocessor will control the data acquisition sampling and conversion in addition to performing the arithmetic function associated with the algorithm.

Having established a tentative block diagram of the instrument, the design continues by addressing relevant characteristics and limitations of each subsystem. These characteristics will then in turn be considered in configuring the final system and program to be executed.

#### 1) The Analog Subsystem

Composed of the analog multiplexer, sample/hold and analog to digital converter, the analog subsystem of Fig. 3-1 affects both system accuracy and throughput rate. The

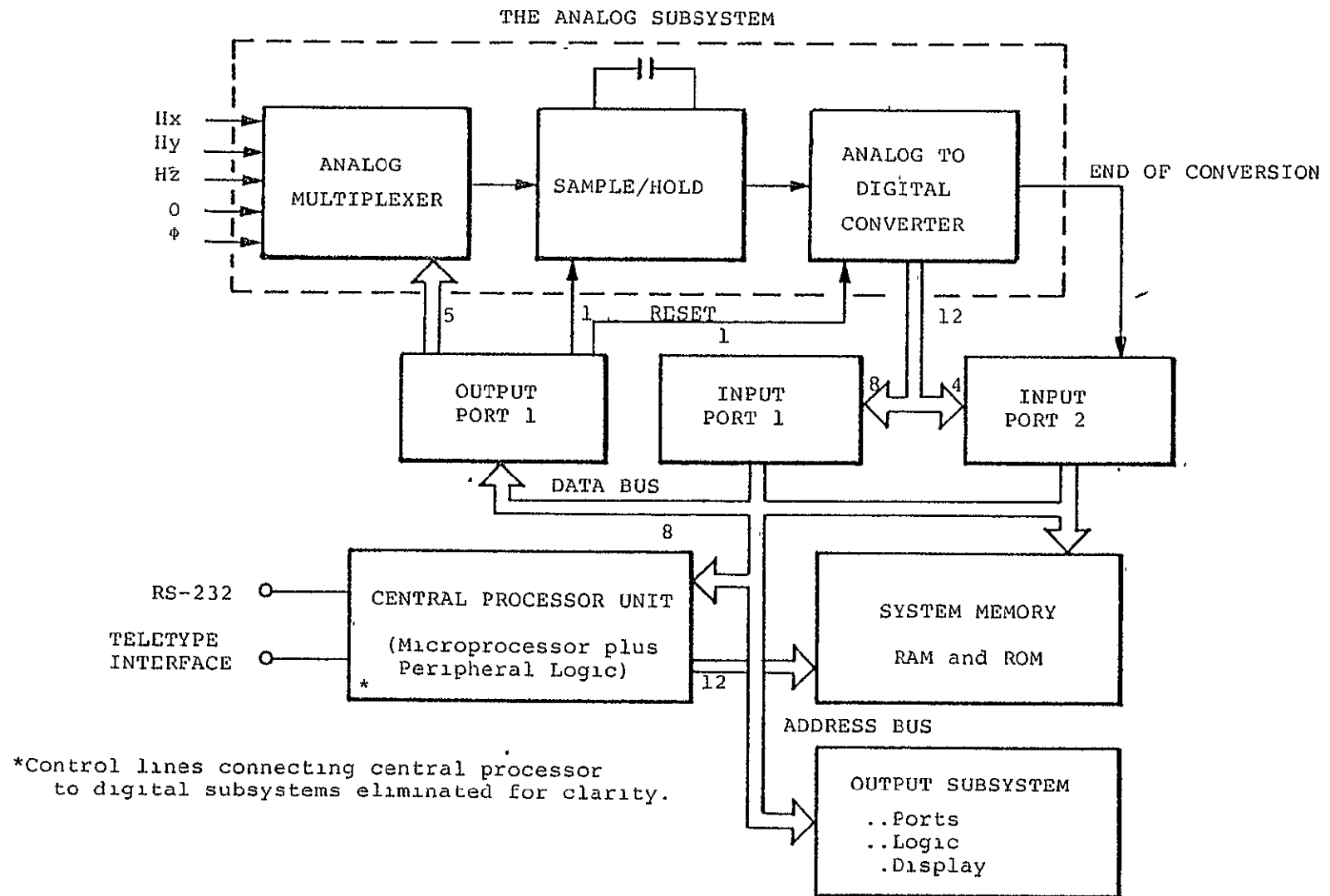


Fig. 3-1. SYSTEM BLOCK DIAGRAM

well-known Shannon theorem [Ref. 3-23, 3-24] on sampling theory defines one of the basic limits on throughput rate stating that the minimum frequency for sampling must be double the highest significant frequency of the signal, including the noise on the signal. This minimum frequency is necessary, the theorem states, if the sampled signal is to contain all of the information needed for undistorted reconstruction. At a lower sampling frequency aliasing can occur<sup>1</sup>. The minimum sampling rate for data to be used in this heading instrument (based on the design goal of Chapter II) then results in a system bandwidth of 30 hertz. The analog signals from each sensor are low pass filtered to reduce frequency content above 60 hertz. A survey of commercially available multiplexers, sample and hold modules and analog to digital convert modules (ADC) [Ref. 3-25 to 3-28] reveals that subsystems with throughput characteristics exceeding the requirements of a system sampled at one second intervals are readily available (pertinent specifications are discussed in more detail in Chapter IV). The limiting parameter determining total system speed performance will then be the execution time of the algorithm (a programming consideration). A further system consideration is the ability to adjust analog system offset and gain. These adjustments are made using variable resistors (trim pots) connected to appropriate leads on the sample and hold and analog to digital converter modules.

## 2) The Central Processing Unit (CPU)

The central processing unit (Fig. 3-2) is composed of the microprocessor (Signetics 2650) supported by peripheral logic elements (Fig. 3-2). Design of this subsystem involved medium

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<sup>1</sup>That is, the sampled data derived from a sine wave of frequency  $f$  sampled at a frequency less than  $2f$  can be fitted with sine waves of a frequency other than  $f$ .

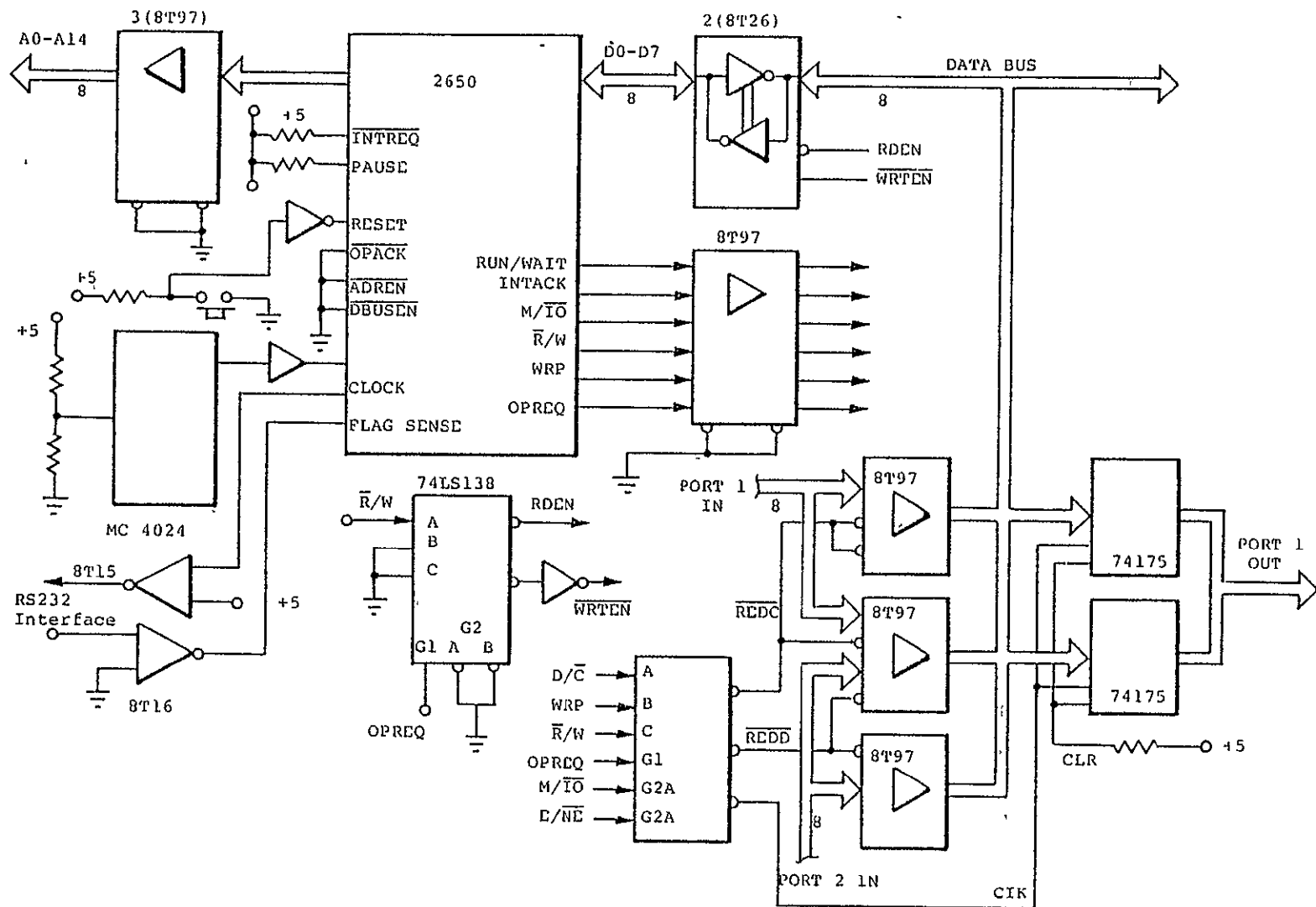


Fig. 3-2. THE CPU SUBSYSTEM



and small scale integrated circuits using well-known [Ref. 3-29 through 3-31] design techniques. To facilitate system development several features were included in the design of the CPU subsystem (features that would not necessarily be required in a production instrument). These include:

- a) System reset, single step and normal run mode operation controlled by switches and logic elements.
- b) An RS-232 teletype interface is included to enable manual intervention and development capability during program development. The program was developed by loading and executing instructions into the random access memory (RAM) under control of the PIPBUG<sup>2</sup> program.

### 3) The Memory Subsystem

The memory subsystem (Fig. 3-3) was organized onto cards each with two thousand byte capability. In this manner system memory could easily be expanded (or reduced) in increments of 2K bytes. The memory chips selected were organized as 256 four bit words and feature pin for pin compatibility with commercially available random access (RAM) and programmable read only memory (PROM) chips. Program segments could then be developed in RAM and finally "burned" into PROM chips for a permanent, nonvolatile operation. In this manner the system development begins with 1K bytes of memory devoted to the resident PIPBUG program (in ROM chips) with the remainder of memory allocated as RAM for both program and scratch pad usage.

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<sup>2</sup>Signetics tradename for the 2650 resident loader and monitor program.

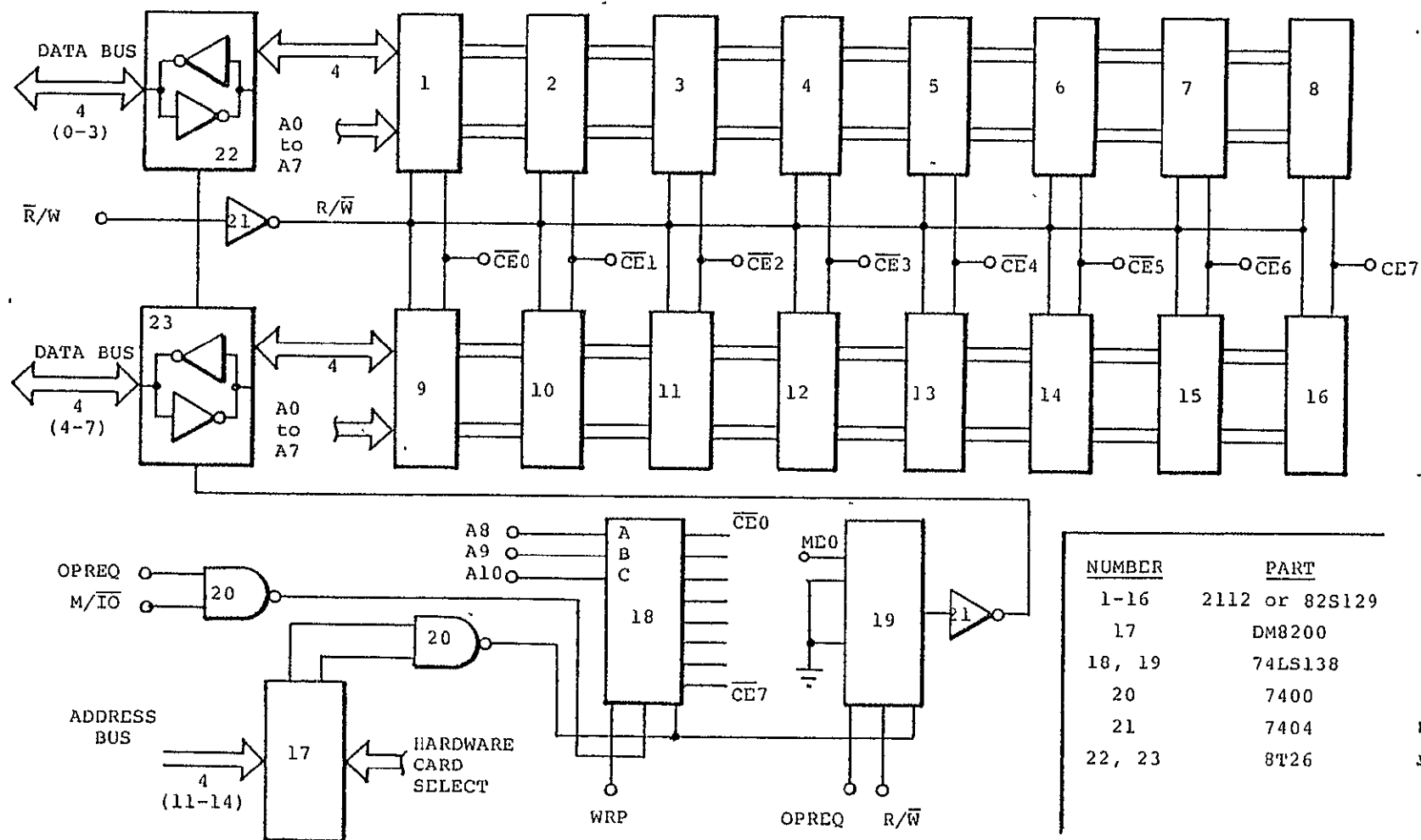


Fig. 3-3. MEMORY SUBSYSTEM (1 Card)

As the program is developed, additional memory is added in increments of 2K bytes per card or 256 bytes on the card. Modifications to the program can be easily made using the PIPBUG program and teletype.

#### 4) The Output Subsystem

For laboratory development the output subsystem of Fig. 3-4 was designed to provide seven segment visual output of the aircraft heading with three significant digits displayed. To expediate the design cycle and to enhance system throughput rate, the outputs were designed as ports with latches and decoder driver functions provided by hardware. In other applications a hardware/software tradeoff could be made with the data decoding and driving implemented using table lookup and multiplexing controlled by the CPU.

### 3-3 SOFTWARE DESIGN CONSIDERATIONS

The general purpose processor selected to implement the CPU was designed to implement programmed logic and to perform conventional computer operations. This heading instrument takes advantage of both areas. Since the instrument is actually a special purpose computer under control of a stored program, the functional specialization resides in the program rather than the hardware logic. Modifications can be made relatively easily, satisfying the flexibility design goal of Chapter II.

Having decided on the tentative hardware structure described in Section 3-2 above, the program development leading to the final listing in Appendix B proceeded as follows:

- 1) Structured flow charts were developed depicting the total system operation as an ordered sequence of operations. Each

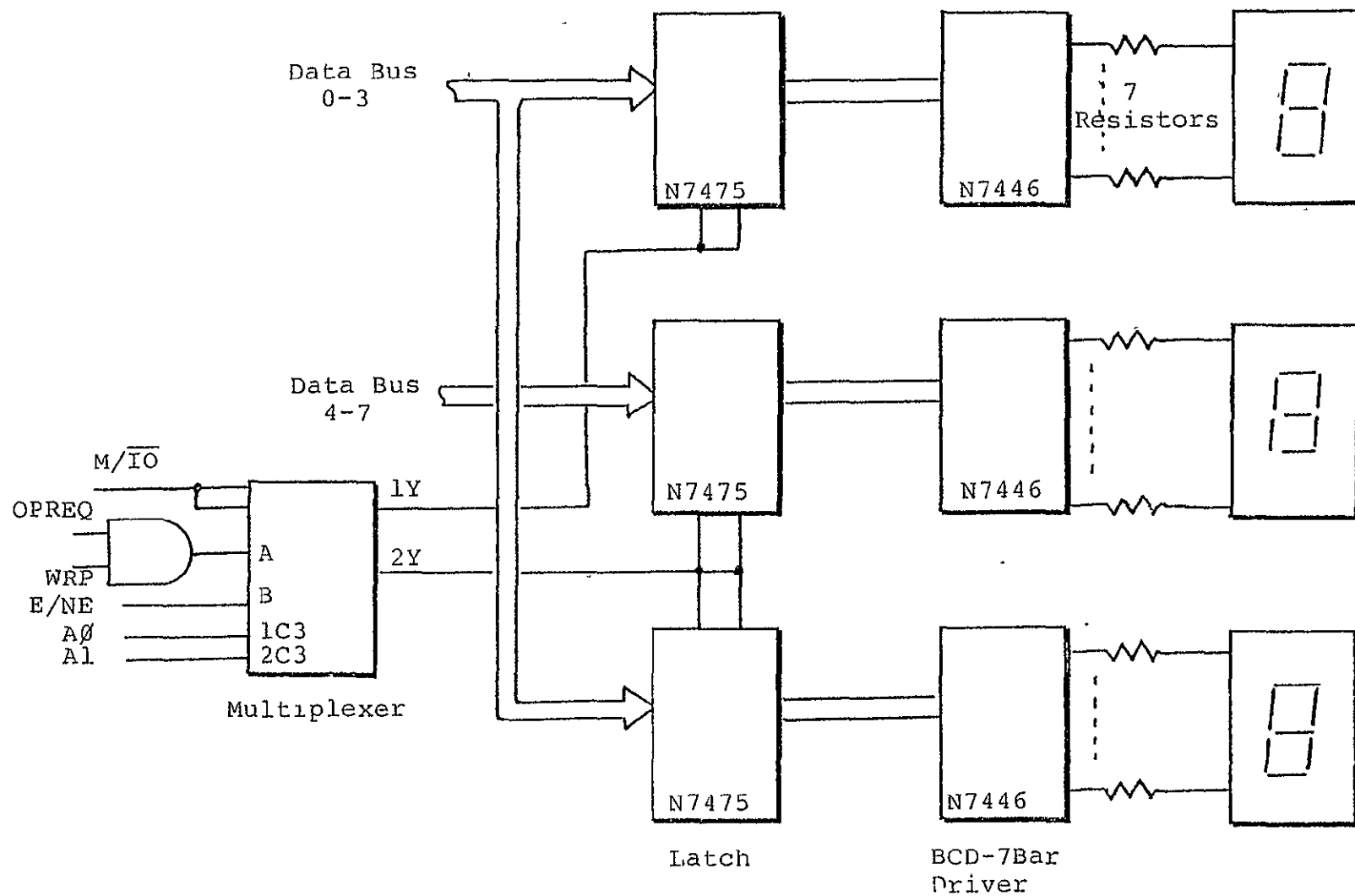


Fig. 3-4 THE OUTPUT SUBSYSTEM

operation is identified as a separate subroutine which in turn can have "nested" subroutines of its own (Fig. 3-5).

2) System accuracy requirements were next investigated (discussed in detail in Chapter IV) to ascertain the precision requirements<sup>3</sup> of the various subroutines.

3) The respective subroutines outlined in 1) above were developed and implemented using a cross assembler program [Ref. 3-32]. Each subroutine was then loaded into the development hardware and "debugged" prior to total program integration. The above program development depicts a top down strategy of program development [Ref. 3-33] and leads to an expedient system development with subroutines being individually developed to yield a modular program construction.

#### 3-4 DESIGN OF SUBROUTINES

The total program consists of an overall system program composed of nested subroutines. The discussion in this section is limited in scope to the design of the more-complex subroutines required to implement the solid state remote magnetic heading algorithm.

##### 1) Subroutine "SAMP" (Fig. 3-6a)

The first portion of this subroutine is dedicated to the control function of selecting an analog channel via the multiplexer, sampling and holding the data, resetting and reading data from the analog to digital converter (ADC). Prior to or during the programming of this section, data fields in

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<sup>3</sup>This step is vital to determine whether the operations outlined in 1) above are to be carried out in a single or multi-precision manner.

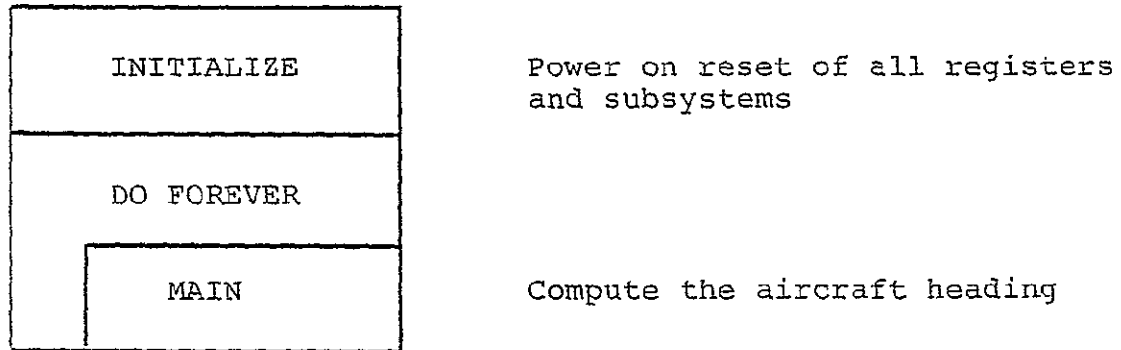


Fig. 3-5a. SYSTEM PROGRAM

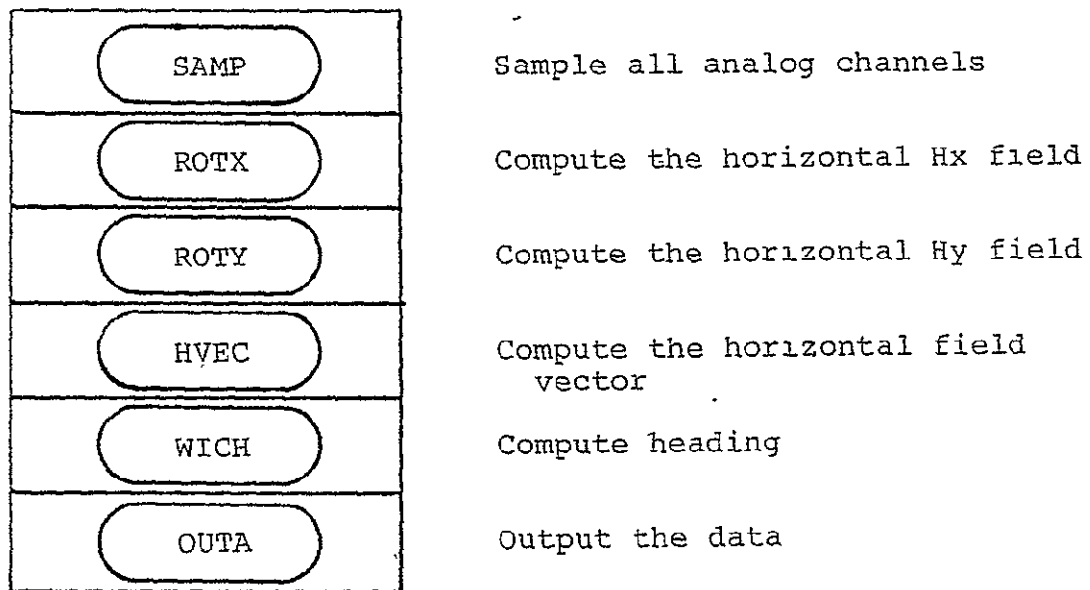


Fig. 3-5b. SUBROUTINE "MAIN"

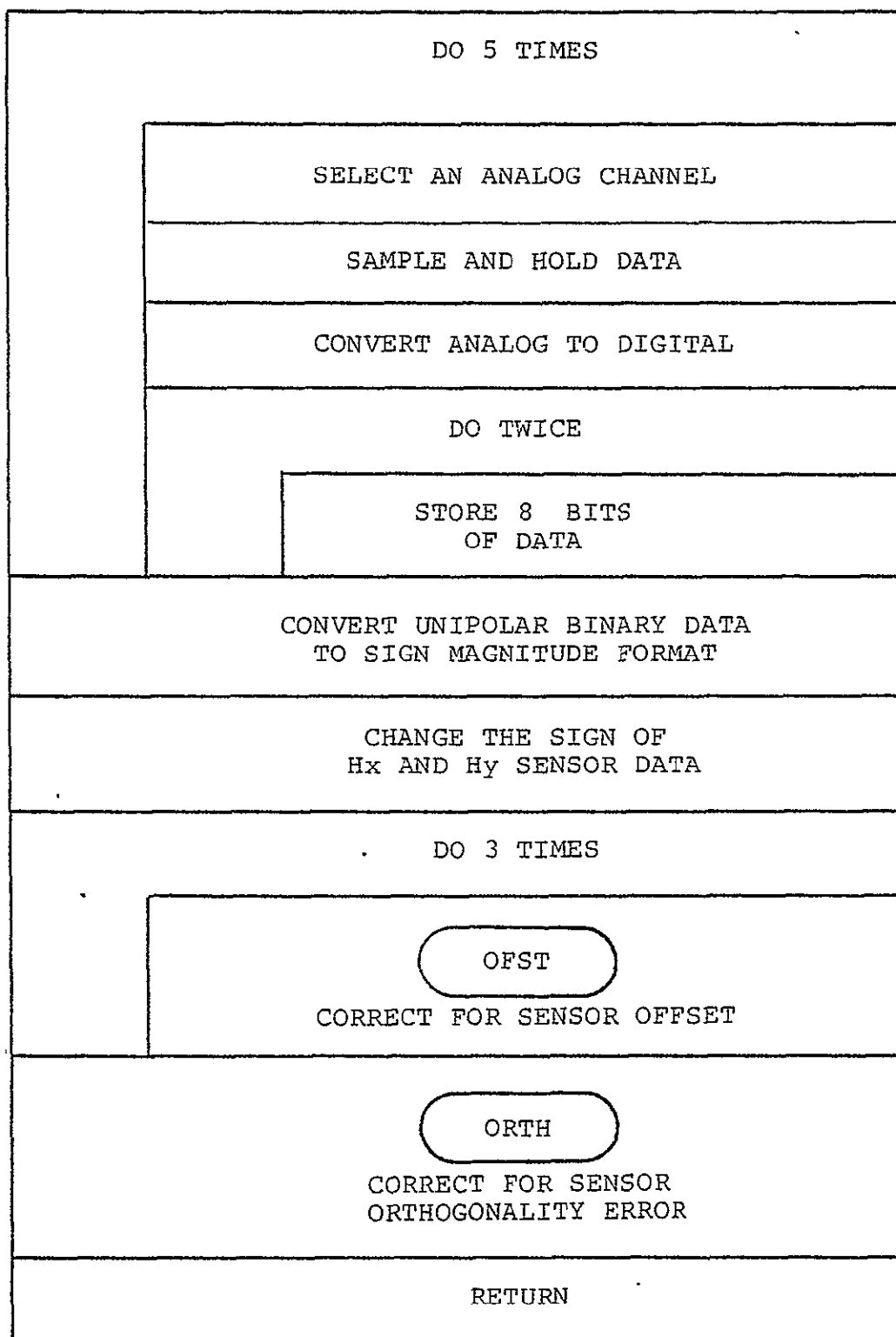


Fig. 3-6a. SUBROUTINE "SAMP"

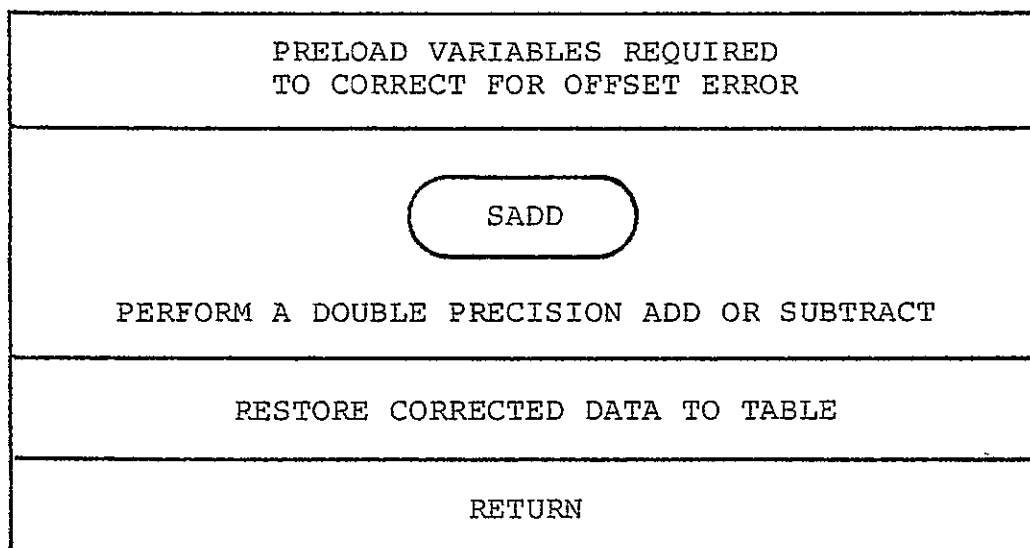


Fig. 3-6b. SUBROUTINE "OFST"

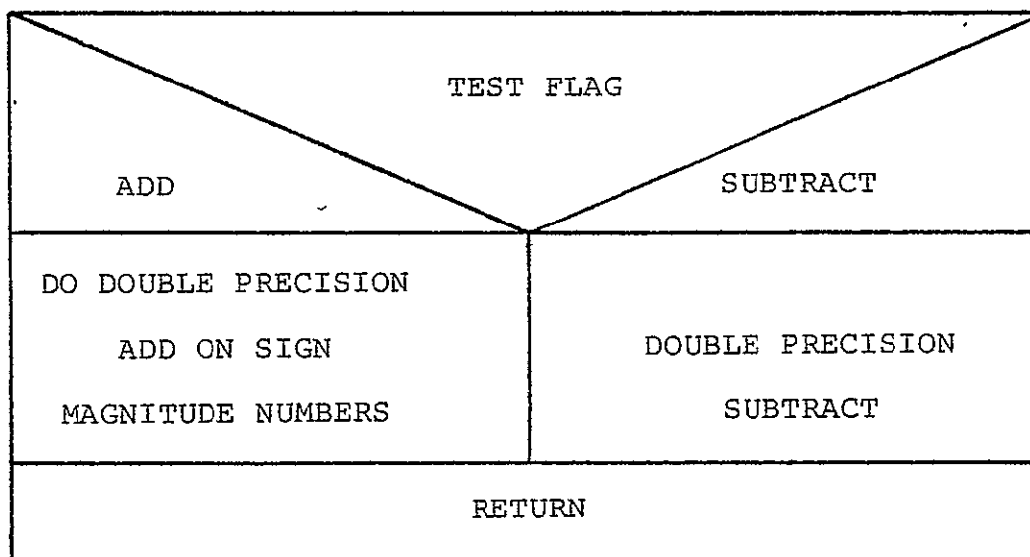


Fig. 3-6c. SUBROUTINE "SADD"



input ports 1 and 2 and output port 1 of Fig. 3-1 are allocated. Control information is then passed to the peripheral module by writing control words to output port 1. Analog to digital converter status and the 12 bit data field are sampled by reading input ports 1 and 2.

Sensor outputs were biased at +2.5 Volts with transfer characteristics as depicted in Fig. 3-7a [Ref. 3-34]. The ADC selected for this laboratory instrument had a binary output data format related to analog input as shown in Fig. 3.7b [Ref. 3-35]. The second function of the sampling subroutine "SAMP" was to convert data from a unipolar binary format to a sign magnitude format. Since the total transfer function from sensor input to ADC output (Fig. 3-7a and b) indicates an offset of 2.5 Volts or 1/2 the ADC output range, the sign magnitude format can be generated as shown in Fig. 3-8.

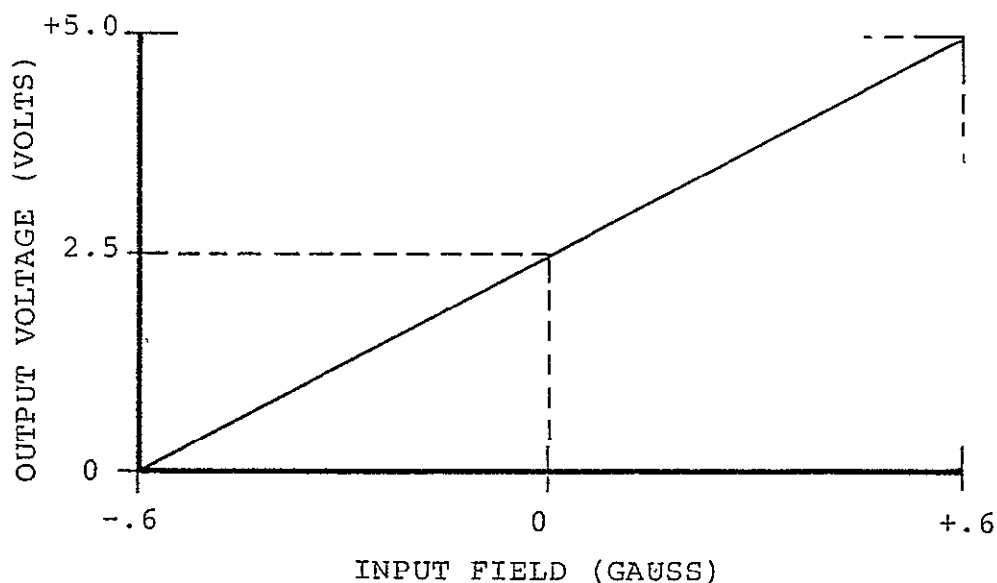


Fig. 3-7a. SENSOR TRANSFER CHARACTERISTIC

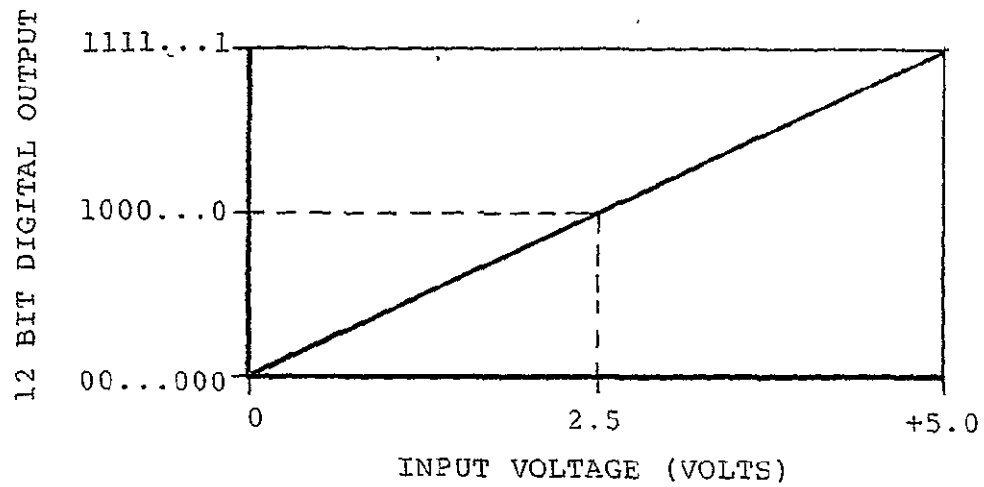


Fig. 3-7b. VDC TRANSFER CHARACTERISTICS

SIGN BIT	
1	0
$x = x_0 + \sum_{i=1}^{11} x_i \cdot 2^i$ <p>(<math>x = x_0 + x^*</math>)</p> <p>Where:</p> <p><math>x_0 = 0</math></p> <p><math>x^* =</math> The 11 least significant bits of the binary number</p>	<p><math>x = x_0 + 2</math>'s complement of the 11 least significant bits</p> <p><math>x_0 = 1</math> (indicating negative quantity)</p>

Fig. 3-8 CONVERSION OF DATA

The third function of the "SAMP" subroutine was to reverse the sign of the Hx and Hy data (to correct a test fixture problem) and to correct for sensor offsets. Although analog subsystem offsets are corrected by adjusting either the sample and hold module or the ADC, the independent sensors themselves have offsets<sup>4</sup>. Offset errors for the laboratory instrument were compensated by determining the offset correction term for each sensor (method described in detail in Chapter V) and then either adding or subtracting the term to the respective data during the sample subroutine. By characterizing the sensor errors<sup>5</sup>, actual datum could be improved further during this step.

The final function of the "SAMP" subroutine was to correct for sensor orthogonality error (subroutine "ORTH"). Although the sensors were physically aligned and specified to have orthogonality characteristic [Ref. 3-34] less than  $\pm 1$  degree relative to the base coordinates, this nonorthogonality contributes appreciably to total system error (see error analysis in Chapter IV). The physical misalignment of the sensors was determined experimentally (Chapter V) and determined to be mainly a misalignment of sensor x in the x-y plane as illustrated in Fig. 3-9.

The actual data measured with the x axis sensor is then related to the true Hx and Hy values as

$$Hx^1 = Hx \cos \epsilon - Hy \sin \epsilon .$$

---

<sup>4</sup>With zero stimulus applied the sensors have a finite nonzero output. This error in the fluxgate magnetometer is a function of temperature, voltage and magnetic remanence in the sensor magnetics [Ref. 3-36].

<sup>5</sup>Sensor characteristics relating the temperature and power supply coefficients of offset error and nonlinearity can be derived empirically.

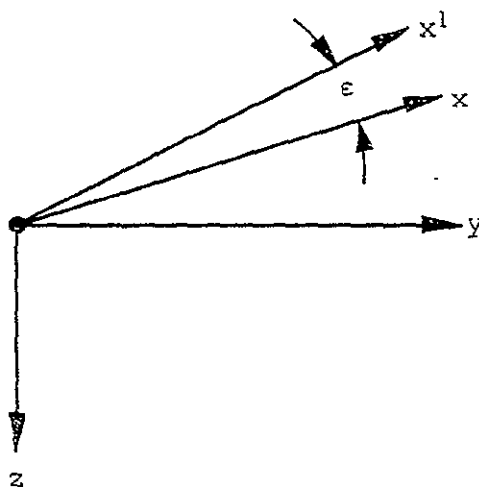


Fig. 3-9 X AXIS NONORTHOGONALITY

Using small angle approximations, we can solve for the desired true value of  $H_x$

$$H_{x^1} \approx H_x - H_y \sin \epsilon \quad (3-1a)$$

$$H_x = H_{x^1} + H_y \sin \epsilon \quad (3-1b)$$

By measuring  $\epsilon$  (Chapter V) and storing the angle as a constant, the x axis data was then restored using equation 3-1b above in subroutine "ORTH".

## 2) Subroutines ROTX and ROTY

These subroutines compute arithmetic values for  $H_{xh}$  and  $H_{yh}$  of equation 2-11b using sign magnitude quantities and table lookup to determine solutions for the transcendental functions. Subroutines "SADD" and "SMPY" are nested and used to perform double precision add and multiply as required.

## 3) Subroutine HVEC

Following computation of the horizontal X and Y axis magnetic vector, the subroutine "MAIN" calls subroutine "HVEC" to compute the square of the horizontal vector. Vectors Hx and Hy are squared by calling subroutine "SQU" then added, yielding  $H(\text{HORIZONTAL})^2$ .

## 4) Subroutine WICH

To compute heading, equation 2-1 (or a similar form) must be solved using the horizontal magnetic field vector and either the x or y axis horizontal field component. Although the square root operation implied in equation 2-1 could be implemented using a numerical technique [Ref. 3-37, 3-38], the computation time is decreased by using a table lookup method. Subroutine "WICH" (Fig. 3-10) compares the absolute magnitude of the two horizontal field vectors Hx and Hy to determine the relative heading of the aircraft<sup>6</sup> with respect to the north-south and east-west axes (Fig. 3-11).

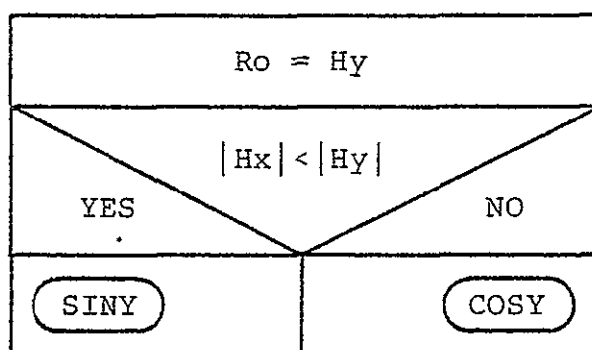


Fig. 3-10. SUBROUTINE "WICH"

<sup>6</sup>If  $|Hx| < |Hy|$ , then an equation similar in form to 2-1a must be used.

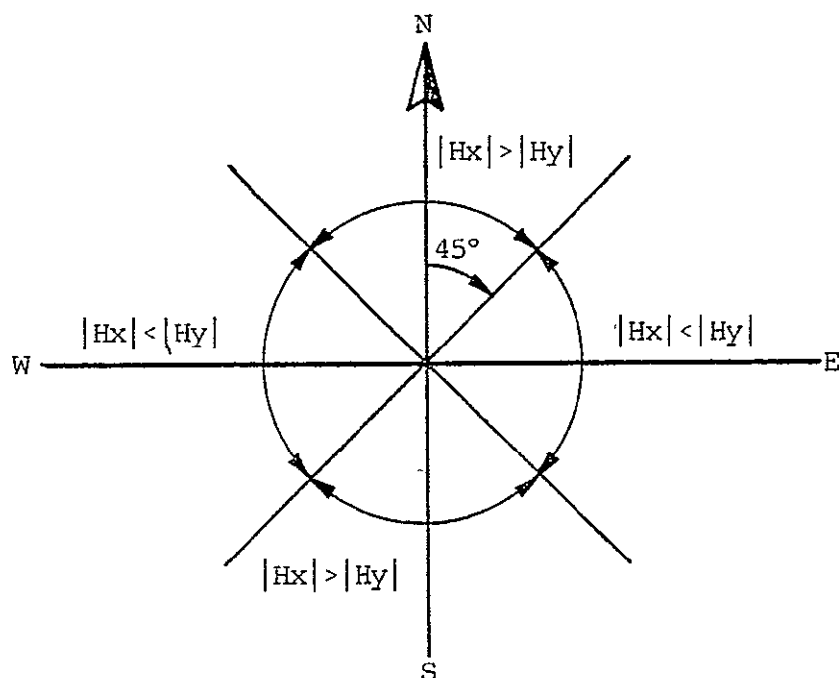


Fig. 3-11. MAGNITUDES OF  $H_x$  AND  $H_y$  RELATED AIRCRAFT HEADING

5) Subroutines COSY and SINY (Fig. 3-12, 3-13)

Depending on the relative absolute magnitudes of  $H_x$  and  $H_y$ , either "COSY" or "SINY" is called to compute aircraft heading. These subroutines invoke subroutine "DIVI" to form the quotient of the axis vector squared and the horizontal field vector squared (a double precision operation). Subroutine "ANGL" is then called to perform an associative table lookup operation using successive approximation and interpolation to complete the inverse cos squared operation. The double precision binary quantity is then converted to three digit binary coded decimal format (BCD) prior to computation of aircraft heading (subroutine "HDG").

The subroutine "SINY" of Fig. 3-12 includes a subtraction of the computed angle from 90 degrees following conversion to BCD format. This operation ensures that the angle passed to

Ro, Rl = Hx <sup>2</sup>	
DIVI	A = Hx <sup>2</sup> /Hh <sup>2</sup>
ANGL	B = arcos <sup>2</sup> (A)
BCDA	Convert B to BCD Format
HDG	Compute Heading
RETURN	

Fig. 3-12. SUBROUTINE "COSY"

Ro, Rl = Hy <sup>2</sup>	
DIVI	A = Hy <sup>2</sup> /Hh <sup>2</sup>
ANGL	B = arcos <sup>2</sup> (A)
BCDA	Convert B to BCD Format
B = 90 - B	
HDG	Compute Heading
RETURN	

Fig. 3-13. SUBROUTINE "SINY"

the calling subroutine upon exiting either "SINY" or "COSY" is an aircraft heading angle relating sensor x to the north-south axis.

6) Subroutine HDG (Fig. 3-14)

The function of this subroutine is to compute aircraft heading having established the angle between the x axis sensor and the north-south geodetic axis. Determination of the actual heading is accomplished by comparing the signs of both the x and y axis horizontal vectors prior to computing heading (Fig. 3-15). It should be noted that all of the preceding computations leading to horizontal vector data were on sign magnitude quantities preserving the correct horizontal vector polarities<sup>7</sup>.

### 3-5 CONCLUSIONS

This chapter has outlined the practical aspects of designing an instrument to evaluate both the heading algorithms and solid state magnetic indicator proposed in previous chapters. The chapter outlined a design approach that can be used to implement a microprocessor based instrument. In particular, the need to consider the total system hardware requirements while simultaneously considering the programming requirements was identified. Design proceeded by outlining a system block diagram (Fig. 3-1) with major subsystems considered. The instrument required a special purpose computer with an analog subsystem to sample and digitize five sensor signals. Timing and control of the analog subsystem plus digital processing of data was controlled by a microprocessor based central processing unit (CPU). Memory for permanent storage of

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<sup>7</sup>It is possible at certain attitudes to require sign reversals when computing horizontal vectors.



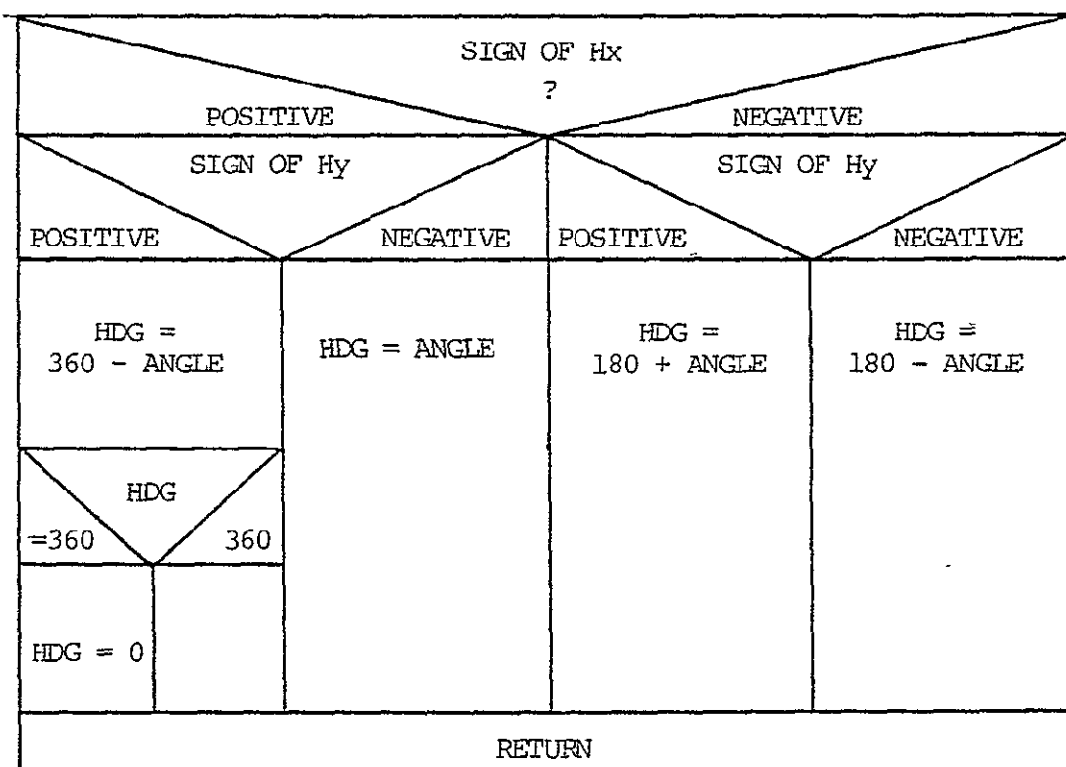


Fig. 3-14. SUBROUTINE "HDG"

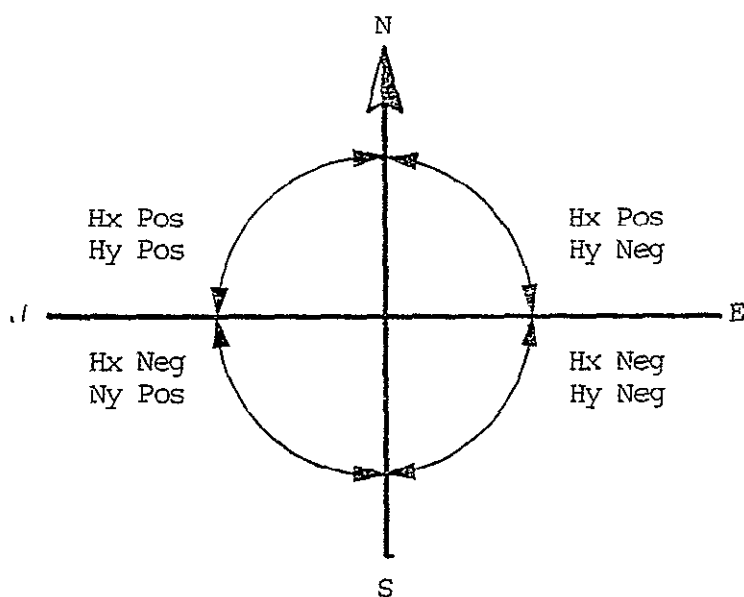


Fig. 3-15. POLARITIES OF HORIZONTAL VECTORS RELATED TO AIRCRAFT HEADING

instructions and temporary storage of data was implemented using memory chips organized on cards with 2048 byte capacity. The particular memory chips selected feature pin compatibility<sup>8</sup> with both read only and volatile random access versions. System inputs consisted of sensor signals from a three axis solid state fluxgate magnetometer plus two analog signals simulating gyroscope outputs. System outputs consist of visual seven segment readout displaying computed heading. In addition, an RS-232 teletype interface was provided to facilitate system development and experimentation.

By identifying the total system in block diagram form at the very beginning, the role and requirements of each subsystem as well as the supporting software were identified. The design then evolved on a modular basis with each subsystem and its supporting program developed in parallel. In this manner pin assignments for input/output ports and critical timing requirements that involved both hardware and software consideration were handled efficiently. By outlining the program requirements in flow chart form (analogous to the block diagram of the hardware subsystem), subroutines were identified facilitating a modular program development. Where possible, subroutines were shared in a nested manner avoiding replication of programming and waste of memory.

Details of error analysis and calculation of overall system throughput rate were deferred to Chapter IV. It was pointed out however, that errors induced by imprecision of data plus truncation and roundoff during processing of the algorithm were to be considered early in the design phase. These data were required to select the sensors and the analog to digital

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<sup>8</sup>Memory integrated circuit (IC) devices of both types can be used in the same mechanical sockets with actual chip type being used transparent to the remainder of the system.

converter as well as to design the supportive software for the analog subsystem. In addition, the data precision requirements were necessary prior to programming the algorithm<sup>9</sup>.

By incorporating a microprocessor as the main CPU element, considerable sophistication in both control and computing performance was achieved. The overall system was designed relatively quickly, provided a convenient laboratory instrument for evaluation of the proposed algorithms and featured inherent flexibility.

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<sup>9</sup>Some of the subroutines required double precision manipulations to maintain overall system accuracy.

## CHAPTER IV

## HEADING INSTRUMENT ERROR ANALYSIS

## 4-1 INTRODUCTION

The heading instrument designed to evaluate the heading and solid state remote magnetic indicator algorithms is prone to error from many sources. These errors will accumulate and degrade the accuracy of aircraft heading or yaw angle computations. This chapter addresses the various error sources to determine their relative magnitudes and effects on the overall computation.

Prior to beginning the hardware design of the microprocessor based instrument many of these potential error sources were considered. Their effects were considered in establishing parameters such as word lengths, A/D converter precision, computation speeds, sampling rates, magnetometer sensor accuracies, system noise tolerance, etc. As the design of the microprocessor based system evolved, the error analysis refined. Ultimately, important limitations in instrument design and operation were identified by combined error analysis and empirical data. By carefully analyzing the source and extent of the limiting parameters (such as sensor offset and non-orthogonality), the magnitude of errors unique to this laboratory sensor array were identified. Specialized software was then added (with empirically derived constants) to correct for the otherwise limiting sensor irregularities improving the total system performance.

In this manner, it is apparent that error analysis is an integral part of instrument design. Not only are important parameters identified early in the design cycle (prior to system block diagram development), but shortcomings in conventional

sensors can be improved by judicial application of error correcting algorithms. In this case, data constants were determined after the final instrument became operational. The sensor peculiarities were analyzed empirically using the instrument itself.

The chapter begins by first identifying and carefully analyzing potential error sources in the sensors. This analysis is followed by a similar consideration of errors originating in the analog subsystem. Processing errors that originate due to the finite word length and precision of the microprocessor along with the effects of simplifications made to the algorithms are finally analyzed. The chapter then concludes with a summary of measurement errors, a sample error analysis, a comparison of predicted to measured error and a summary.

#### 4-2 SENSOR ERRORS

The heading computation algorithm employing the remote magnetic indicator (Chapter II) is prone to error proportional to both fluxgate magnetometer sensor and gyroscope measurement errors. Errors inherent in the fluxgate magnetometer are summarized on the data sheet [Ref. 3-34]. Since the experimentation employed simulated gyroscope sensors with voltage levels accurately represented, the analysis of sensor errors will assume ideal gyroscope sensors to predict experimental data.

##### A) Sensor Offset Error

Magnetometer sensors exhibit error caused by both electronic and magnetic phenomena. Errors in the Develco sensors were outlined by Workentine [Ref. 4-1]. These offset errors are induced in the Develco sensors by both electronic offset voltages and currents in the respective sensor electronics and

by residual magnetic fields in the magnetic mass of the sensor assemblies. Although the physical and electronic design attempts to reduce offset error, a finite non-zero output can exist when a zero input is applied.

Offset error for each sensor in the Develco model 9200C three axis magnetometer assembly is specified [Ref. 3-34] as "Zero Field Bias +2.5 Volts  $\pm 1.0\%$ ". This offset translates into a worst case maximum error voltage of

$$E_{\text{OFFSET}} = \pm(2.5\text{V} \times 0.01) = \pm 25\text{mV}$$

Since the offset error is sensor dependent, correction cannot be made at a single physical point (as for analog subsystem offsets described in Section 4-3). Corrections can however be made to the measured data by simply adding or subtracting a constant equal to the offset magnitude following each data measurement<sup>1</sup>.

Offset values for each sensor used in the experiment were obtained by rotating the sensor into alignment with earth's magnetic field vector to measure both positive and negative maximum values. The difference in magnetic measurement (assuming negligible analog subsystem error) is related to system offset error composed of sensor electronic and sensor plus test fixture induced magnetic offset error. The actual offset error can be calculated using these two measurements

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<sup>1</sup>Offset corrections were made in the sample subroutine "SAMP" illustrated in Fig. 3-6a.

$$\begin{aligned}
 |E_{\max}| &= E_f + E_o \\
 |E_{\min}| &= E_f - E_o \\
 |E_{\max}| - |E_{\min}| &= (E_f + E_o) - (E_f - E_o) = 2 E_o \\
 E_o &= (1/2) (|E_{\max}| - |E_{\min}|)
 \end{aligned}$$

where

$E_{\max}$  = The maximum positive voltage recorded when the sensor aligns with earth's field vector.

$E_{\min}$  = The maximum negative voltage recorded when the sensor aligns 180° with earth's field vector.

$E_f$  = The magnitude of earth's magnetic vector represented in volts.

$E_o$  = The sensor offset voltage due to both electronic and magnetic phenomena

Data recorded during x, y and z axis offset measurements as described above are recorded in Table 4-1. Since the offset error is a function of sensor magnetic permeability, the actual offset value will vary with time depending on induced magnetic fields<sup>2</sup>.

Final offset correction values were determined by rotating two sensors in the horizontal plane around the third vertical axis and measuring offsets in two sensors at a time. Recorded data for each sensor was previously corrected for orthogonality error by the sample subroutine "SAMP" (discussion

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<sup>2</sup>For example, magnetized screwdrivers or other tools used near the sensor will alter the residual magnetic field.

SENSOR AXIS	DATA RECORDED (HEXADECIMAL)		OFFSET (HEXADECIMAL PLUS SIGN)
	Emax	Emin	
X	628	E78	+40
Y	640	E5D	-15
Z	637	E68	-25

Table 4-1    OFFSET DATA DERIVED BY MEASURING  
EARTH'S FIELD



of this correction follows in Section 4-2B). Data recorded in this manner, appears in Tables 4-2 and 4-3. Final correction terms for correcting sensor offset error were calculated using these data. Offset terms to be added or subtracted from respective data channels are tabulated in Table 4-4.

By correcting system offset errors in this manner, the effective error contribution can be reduced appreciably (see final data discussion Chapter V). For a flight instrument, sensor offset characteristics as a function of temperature variation and supply voltage can be derived empirically and appropriate offset corrections made by computing the value of the correction term variable. Magnetically induced offsets can be reduced by degaussing the sensor assembly periodically.

#### B) Axis Alignment Errors

The error specification of [Ref. 3-34] indicates that the maximum axis alignment error is  $\pm 1$  degree relative to base referenced coordinates. This error results in sensor directional uncertainty as illustrated in Fig. 4-1. Each sensor is located within a right circular cone with axis along the true sensor axis and vertex at the common sensor origin. Although this alignment uncertainty contributes no error in determining the total magnetic vector

$$\bar{H} = (\bar{H}_x^2 + \bar{H}_y^2 + \bar{H}_z^2)^{\frac{1}{2}},$$

there is considerable uncertainty in attempting to resolve the true magnetic field component along any axis of the reference coordinate system. This alignment uncertainty of magnetic sensors limits system performance of conventional field direction measuring apparatus [Ref. 4-1].

Protractor Heading Measurement (Degrees)	Data Measured		Protractor Heading Measurement (Degrees)	Data Measured		Error Due To Offset	
	Hx	Hy		Hx	Hy	X	Y
		(Units)			(Units)	(Units)	
0	8	-759	180	83	720	75	-39
345	199	-743	165	-112	704	87	-39
330	383	-677	150	-292	638	91	-39
315	540	-569	135	-448	527	92	-42
300	666	-421	120	-573	381	93	-40
285	749	-249	105	-658	206	91	-43
270	783	- 62	90	-696	14	87	-48
255	770	132	75	-681	-179	89	-47
240	706	314	60	-616	-362	90	-48
225	597	473	45	-504	-521	93	-48
210	449	601	30	-359	-645	90	-44
195	273	687					
180	(DATA UNAVAILABLE DUE TO TEXT FIXTURE LIMITATION)						
165							
TOTAL OFFSETS						978	477
AVERAGE OFFSETS						88.9	43.4

Table 4-2 X AND Y AXIS ERROR MEASURED BY ROTATING  
X, Y AROUND Z IN THE HORIZONTAL PLANE

Protractor Heading Measurement (Degrees)	Hz Data Measured (Units)	Protractor Heading Measurement (Degrees)	Hz Data Measured (Units)	Offset Error (Units)
0.5	0	180.5	-56	-56
315.5	-527	135.5	469	-58
270.5	-763	90.5	707	-56
225.5	-571	45.5	513	-58
TOTAL OFFSET				-228
AVERAGE OFFSET				-57

Table 4-3 Z AXIS OFFSET ERROR MEASURED BY ROTATING  
THE Z AXIS AROUND THE VERTICAL X AXIS

Sensor Axis	Total Average Offset (Units)	Required Correction	Amount of Correction Decimal	Binary	Hex
X	88.9	Subtraction	45	00101101	02D0
Y	43.4	Addition	22	00010110	0160
Z	47.0	Addition	29	00011101	01D0

Table 4-4 OFFSET CORRECTION VALUES

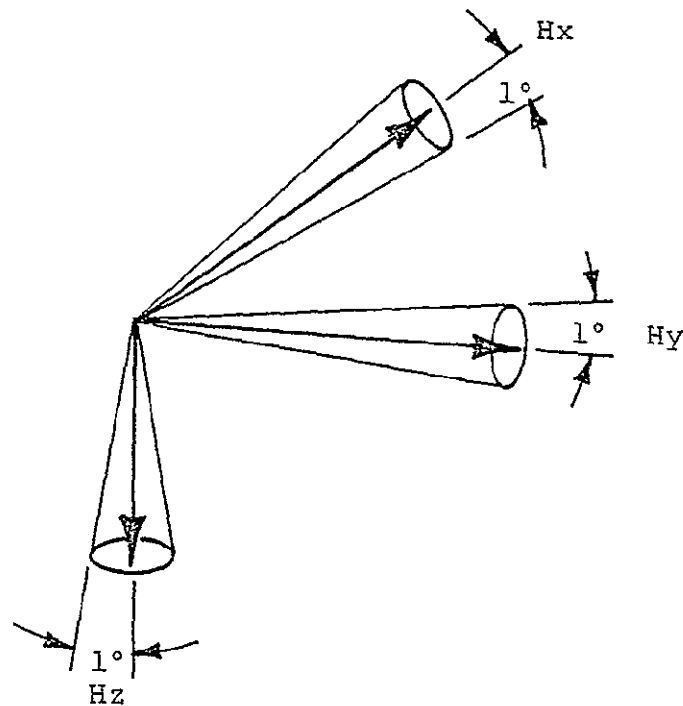


Fig. 4-1 SENSOR ALIGNMENT UNCERTAINTY

Although this error source can be reduced by physically aligning the sensors more accurately during assembly, cost of the sensors increases. Ultimately, directionality of the magnetic sensors becomes a function of the physical sensor itself and more accurate sensors are required as pointed out by Gise [Ref. 4-2]. A heading system that tolerates sensor misalignment is therefore a very desirable alternative to requiring precise alignment or more elaborate sensors.

During assembly of the Develco fluxgate magnetometer sensor array, sensor misalignment is determined by using earth's magnetic field and a precision mechanical rotation assembly. A sensor (assume the X axis) is aligned with earth's magnetic

vector by positioning the sensor to maximize electrical output<sup>3</sup>. One of the other sensors (assume the y axis) is aligned with the rotation axis of the precision calibration assembly (Fig. 4-2) and perpendicular to the first by rotating the sensor array around the second sensor axis (y axis in this case) and adjusting its relative position until a null output is achieved at all rotation angles. Mechanical orthogonality of the sensors is then limited only by the mechanical imprecision of the calibration device (orthogonality within  $\pm 0.01$  degrees can be easily achieved in the calibration tool) and by the directional characteristics of the physical sensors.

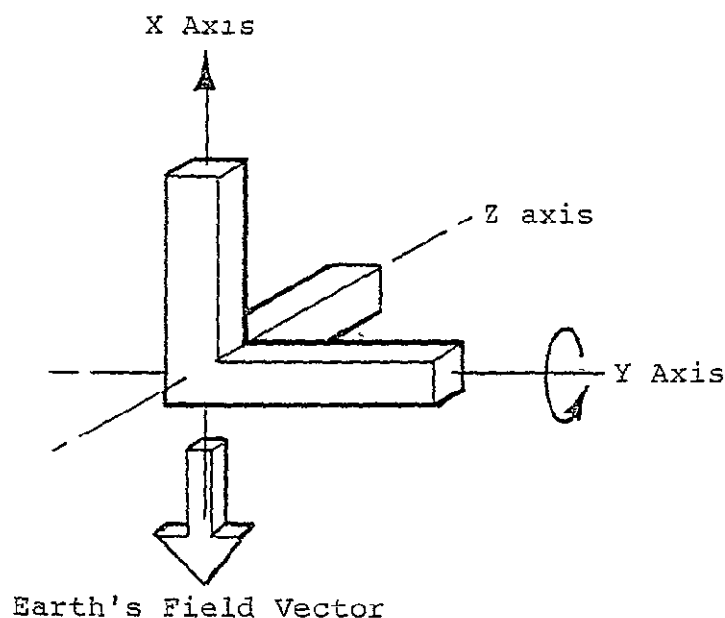


Fig. 4-2 MECHANICAL ORIENTATION OF THE MAGNETOMETER SENSORS DURING CALIBRATION

In addition to functioning as an alignment apparatus, the calibration device described above provides a convenient means

<sup>3</sup>By maximizing or nulling a measurement, the mechanical positioning is a function of only the field and the resolution of the voltage measuring device obviating errors due to physical position measurement.

to characterize sensor assemblies after final assembly adjustments are made. Any misalignment of the second sensor relative to the first results in a coning of the second sensor around the rotation axis<sup>4</sup> with a sinusoidal output voltage that is a function of total earth's magnetic field and axis alignment error. The peak to peak voltage resulting from sensor coning is recorded during the final alignment test and made available to sensor purchasers. Coning voltages developed for the sensor assembly used with this experiment were obtained from Develco [Ref. 4-3] and are recorded in Table 4-5. Sensor misalignment for each axis can be derived using additional data provided by Develco along with additional empirical data derived by experimentation.

The total ambient magnetic field at the Develco laboratory is measured using the three sensors (applying equation 4-1) and is supplied as digital data. In our case, the total field measured was 15.73 units or

$$\frac{1573 \text{ units}}{2048 \text{ units Full Scale (F.S.)}} \times 60,000 \text{ gamma F.S.} = 46,084 \text{ gamma } (\gamma)$$

$$\text{Sensitivity of the sensor} = \frac{2.5 \text{ Volts F.S.}}{60,000 \gamma \text{ F.S.}} = 42 \mu\text{Volts}/\gamma$$

Considering the X axis sensor, coning resulted in a signal of 38 mV peak to peak (or 19mV peak). Misalignment of the X axis sensor from the Y-Z plane can then be calculated as

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<sup>4</sup>Assume that the first axis is initially adjusted for maximum output to align it with earth's field and the rotation axis is perpendicular to the field.

## SENSOR ASSEMBLY NO. S/N 1043-013

Rotation Axis	Coning Voltage (Peak-Peak mV)	Orthogonality Error (Degrees)
X	38	0.57
Y	8	≈0
Z	51	0.76

Table 4-5 MAGNETOMETER ORTHOGONALITY MEASUREMENTS

Peak Signal = 19 mV or 456 gamma angular misalignment

$$\epsilon_x = \sin^{-1} \frac{456}{46,084}$$

$$\epsilon_x = 0.57 \text{ degrees}$$

Similarly, the Y and Z axis have misalignment errors of  $\epsilon_y = 0$  and  $\epsilon_z = 0.76$  degrees with respect to the X-Z and X-Y planes respectively (sensor orthogonality errors are tabulated in Table 4-5).

Having established that sensor orthogonality errors exist, the remaining task is to identify the direction that the sensor axis points relative to the other two sensor axes. Since the Y axis has relatively little orthogonality error, it will be assumed to be perpendicular to the X-Z plane. In addition, since the Hz data enters into the algorithm in a second order manner relative to the Hx and Hy measured data, correction and characterization of the Hx sensor was considered to be of primary concern. Orientation of the X axis sensor relative to the Y and Z axes was determined empirically.

Angular position of the X axis sensor can be described using the error angles  $\epsilon_{xy}$  and  $\epsilon_{xz}$  as delineated in Fig. 4-3. Characterization of sensor orthogonality error in terms of these two angles would enable algorithmic corrections of measured data.

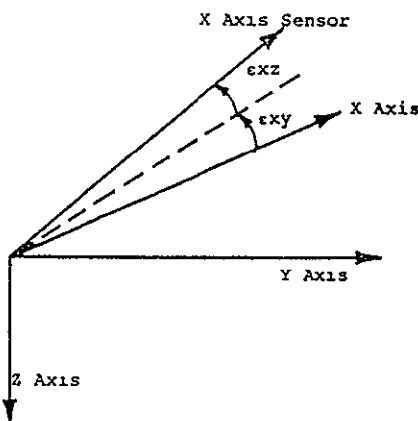


Fig. 4-3 X AXIS SENSOR ORIENTATION



# 1) Empirical Determination of $\epsilon_{xz}$

The angle  $\epsilon_{xz}$  (angle between the x axis sensor and the z axis of the geodetic coordinate system) was determined in several steps using the test apparatus described in Chapter V.

- i) The x and y sensors were oriented in the horizontal plane with the z axis sensor vertical downward.
- ii) The x and y sensors were rotated around the z axis with magnetic data measurements (corrected for sensor offset error as described in Section 4-2A) recorded in Table 4-6 for incremental rotation angles.
- iii) The total horizontal field at each angular position was calculated

$$H_{ht} = (H_x^2 + H_y^2)^{\frac{1}{2}}.$$

- iv) Average horizontal field  $H_{av}$  was computed by averaging the results of iii) above.
- v) The horizontal field deviation  $H_d$  was computed for each angular position; tabulated in Table 4-6 and plotted on Fig. 4-4.

$$H_d = (H_{av} - H_{ht})$$

The horizontal field deviation or error (as shown on Fig. 4-4) was now examined. An angular error  $\epsilon_{xz}$  should cause the horizontal field error curve to peak at 90 and 180 degrees. Since this obviously was not the case, it was concluded that major error in x axis orthogonality was due to the component  $\epsilon_{xy}$ .

Physical* Heading (Degrees)	Displayed** Heading (Degrees)	Measured Data (Units)		Total Computed Horizontal Field (Hht) (Units)	Hd (Hav-Hht) (Units)
		Hx	Hy		
355	90	12	-727	727	-3
335	70	258	-682	729	-1
315	50	477	-555	732	2
295	30	637	-362	733	3
275	10	720	-127	731	1
255	350	718	127	729	-1
235	330	628	363	725	-5
215	310	464	569	734	4
195	290	242	687	728	-2
175	270	-10	731	731	1
155	250	-258	686	733	3
135	230	-476	557	733	3
115	210	-635	362	731	1
95	190	-721	123	731	1
75	170	-718	-133	730	0
55	150	-628	-371	729	-1
35	130	-460	-565	729	-1

Total Hht = 12415

Average (Hav) = 730

\*Measured using a protractor on the test apparatus.

\*\*Computed and displayed digitally by the instrument.

Table 4-6 MEASUREMENT OF HORIZONTAL FIELD

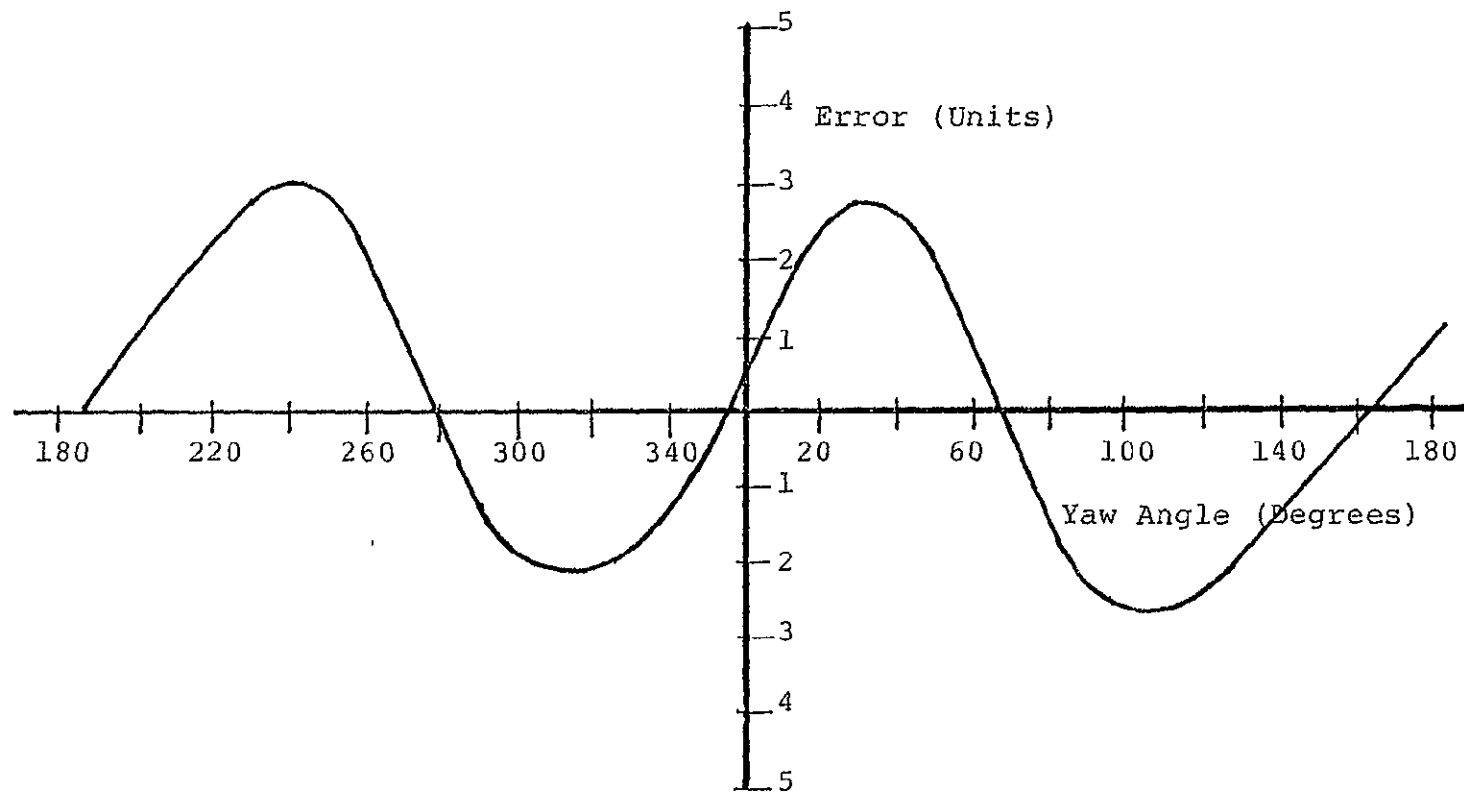


Fig. 4-4 DEVIATIONS OF THE HORIZONTAL FIELD MEASUREMENT  
FROM THE MEAN

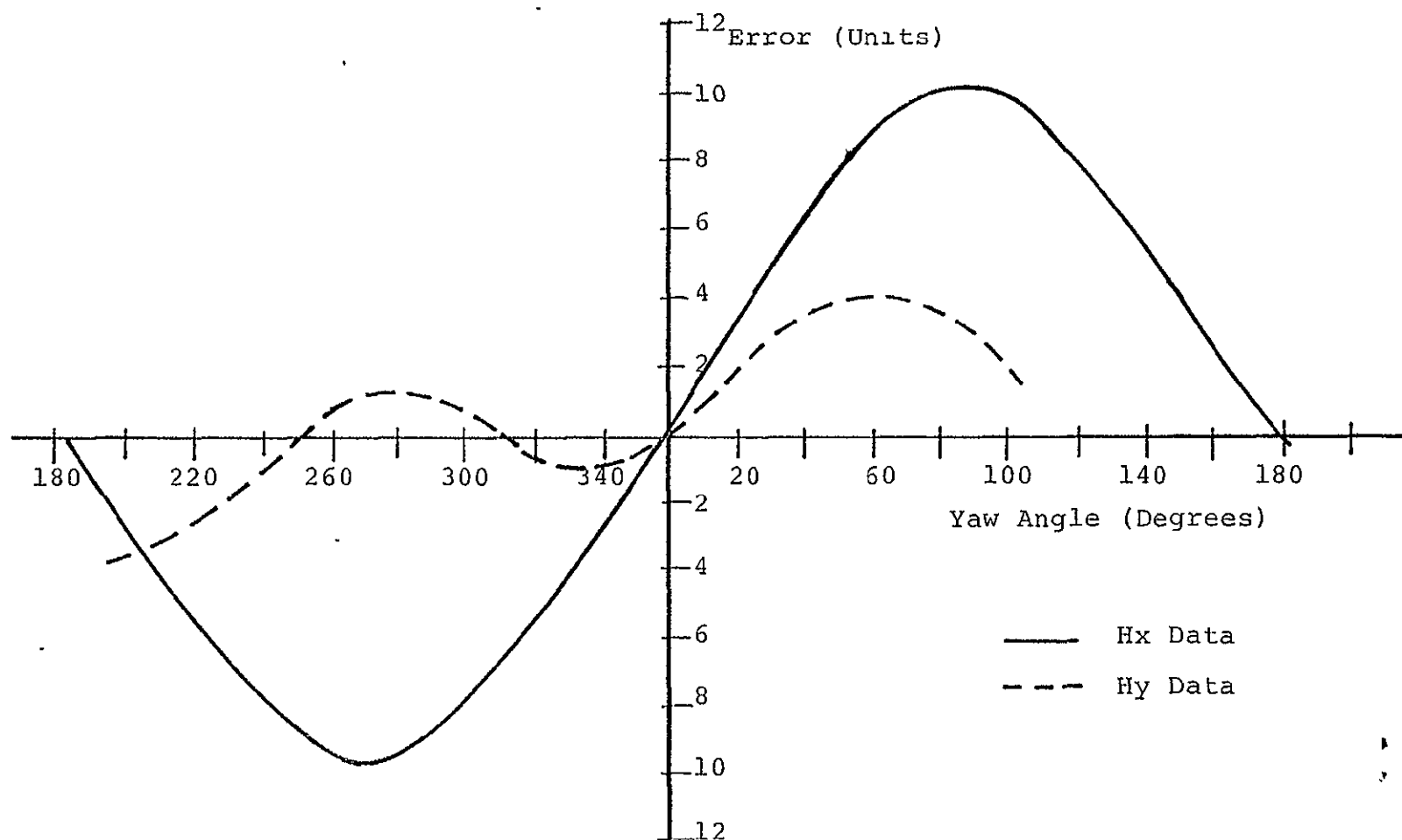


Fig. 4-5 DEVIATION OF Hx AND Hy DATA FROM THE COMPUTED FIELD COMPONENTS AS A FUNCTION OF YAW

## 2) Empirical Determination of $\epsilon_{xy}$

The angle  $\epsilon_{xy}$  representing x axis sensor misalignment relative to axis y was measured as follows:

Steps i) and ii) above were repeated with the exception that the calculated values for Hx and Hy (Hxc and Hyc respectively) were recorded with measured Hx and Hy data (Hxm and Hym respectively) in Table 4-7. The calculated values were obtained by assuming that the angle  $\epsilon_{xz}$  as determined above was negligible and that the y axis sensor was perpendicular to the x-z plane. With these assumptions, we note that at the heading of zero degrees (extrapolated between display of 10 and 350 degrees of Table 4-7 and Fig. 4-5), there is no error in yaw due to either Hx or Hy. By physically rotating the sensors in fixed intervals from yaw = 0 degrees and noting that the horizontal field  $H_h = 730$  units, we can then compute expected Hx and Hy data at respective yaw orientations.

Physical orientation of the x axis sensor is easily determined by considering orientation at the maximum error excursions. These observations are illustrated in Fig. 4-6. We note that the only possible orientation of the x axis sensor satisfying the data in Fig. 4-5 is that of Fig. 4-6.

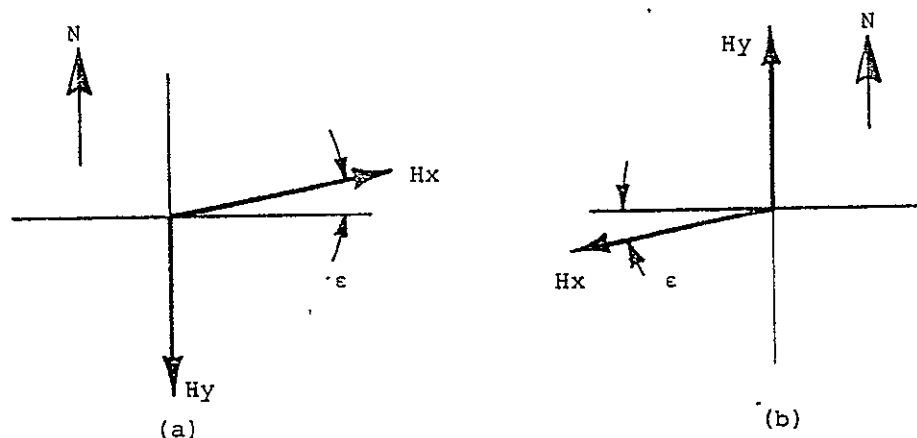


Fig. 4-6 (a) SENSORS ORIENTED AT YAW = +90 degrees  
(b) SENSORS ORIENTED AT YAW = +270 degrees

Physical* Heading (Degrees)	Displayed** Heading (Degrees)	Measured Data (Units)		Computed Data (Units)		Deviation (Units)	
		Hxm	Hym	Hxc	Hyc	(Hxm-Hxc)	(Hym-Hyc)
355	90	12	-727	0	-730	+12	+ 3
335	70	258	-682	250	-686	+ 8	+ 4
315	50	477	-555	469	-559	+ 8	+ 4
295	30	637	-362	632	-365	+ 5	+ 3
275	10	720	-127	719	-127	+ 1	0
255	350	718	127	719	+127	- 1	0
235	330	628	363	632	365	- 4	- 2
215	310	464	569	469	559	- 5	10
195	290	242	687	250	686	- 8	1
175	270	-10	731	0	730	-10	1
155	250	-258	686	-250	686	- 8	0
135	230	-476	557	-469	559	- 7	- 2
115	210	-635	362	-632	365	- 3	- 3
95	190	-721	123	-719	+127	- 2	- 4
75	170	-718	-133	-719	-127	1	- 6
55	150	-628	-371	-632	-365	4	- 6
35	130	-460	-565	-489	-559	9	- 6

\* Measured using a protractor on the test apparatus.

\*\* Computed and displayed digitally by the instrument.

Table 4-7 MEASURED AND COMPUTED Hx AND Hy DATA  
IN THE HORIZONTAL PLANE

Magnitude of the angle  $\epsilon_{xy}$  can be computed as follows using data from Fig. 4-5

Max. delta from Fig. 4-5 = 10 units  
Average horizontal field = 730 units

$$\epsilon_{xy \text{ max}} = \sin^{-1} \frac{10}{730}$$

$$= 0.79 \text{ degrees}$$

We note that the angle of 0.79 degrees is approximately the same as determined by Develco during manufacture of the sensors (Table 4-5). The added error is due to test set inaccuracy.

#### C) Fluxgate Sensor Noise Induced Error

The analog output from the fluxgate sensors can exhibit an error due to signal uncertainty resulting from noise. Although the data sheet [Ref. 3-34] indicates that 5mV peak to peak of ripple can exist on the output, the frequency content centers in the 550 kHz range (driver frequency of the fluxgate magnetometer) and no appreciable ripple<sup>5</sup> exists below 60 Hz (especially when the sensor output is filtered prior to data sampling). The noise specification of less than 1 gamma peak to peak in the 1 Hz bandwidth region is also negligible. In summary, no appreciable error due to noise on the magnetometer signal lines is evident.

#### D) Magnetometer Gain Error

The magnetometer is specified to have gain (sensitivity) of 2.5 Volts/600 milligauss,  $\pm 1\%$  which translates into a maximum signal uncertainty of

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<sup>5</sup>Verbally confirmed by Workentine of Develco [Ref. 4-1].

$$\pm(2.5V \times 0.01) = \pm 25 \text{ mV.}$$

This represents a sensor transfer function of 4.16 Volts/gauss or 0.24 gauss per volt. The uncertainty then can be expressed as

$$\begin{aligned} \pm(0.24 \text{ gauss} \times 0.01) &= \pm 2.4 \text{ milligauss} \\ &= \pm(2.4 \times 10^2) \text{ gamma} \end{aligned}$$

Since this error is not corrected in the laboratory instrument it will be considered in total in the final error analysis. It is worth noting however, that should the magnetometer gain uncertainty be characterized, gain corrections for each sensor could be made during computation by the computer. In addition the error term is proportional to actual signal level applied.

#### E) Magnetometer Linearity Error

D.C. linearity of the magnetometer is specified to be  $\pm 0.5\%$  of signal level. This uncertainty at full scale can be expressed as  $\pm(2.5 \text{ Volts} \times 0.005) = \pm 12.5 \text{ mV}$ . Alternately, linearity error can cause a signal uncertainty of  $\pm 1.2$  milligauss or  $\pm(1.2 \times 10^2)$  gamma. Linearity error is also not corrected during computation and is considered in the final error analysis. By simply characterizing and correcting the linearity characteristics of each sensor, considerable improvement in system accuracy could be achieved.

### 4-3 ANALOG SUBSYSTEM ERROR ANALYSIS

The analog subsystem of the instrument is outlined in block diagram form in Fig. 4-7. This subsystem accepts analog signals from magnetometer and gyroscope transducers, performs a time division multiplexing between the signals and digitizes



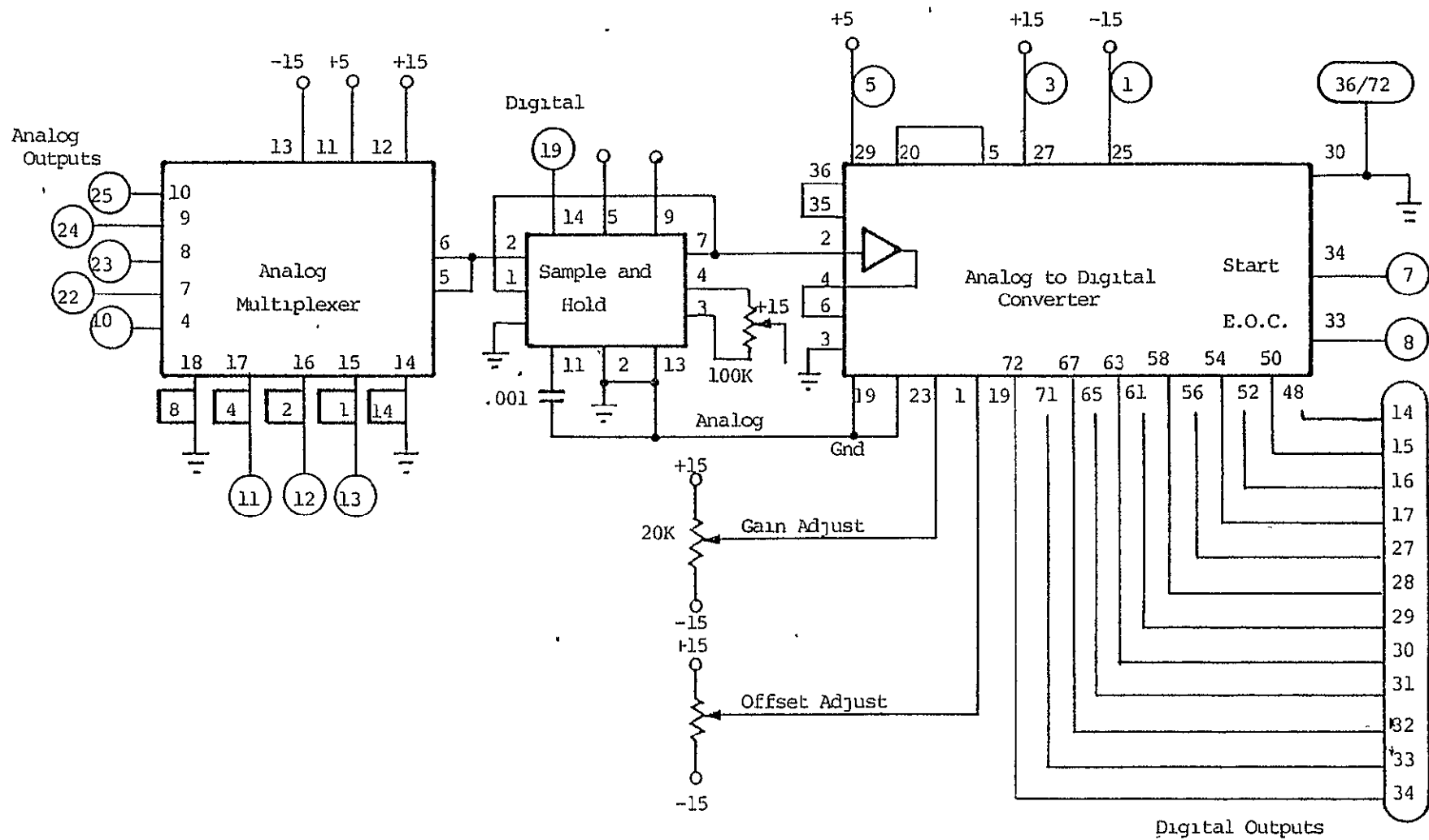


Fig. 4-7 THE ANALOG SUBSYSTEM

the respective signals prior to subsequent processing by the computer. During this data acquisition and conversion process, errors are introduced into each of the signals. This section addresses the potential error sources and computes the respective error contributions to be expected during operation of the instrument.

Although the multiplexer and sample and hold blocks of Fig. 4-7 could be eliminated (eliminating possible error sources) by digitizing each signal with a unique analog to digital converter, it can be shown that such a system would be expensive and difficult to implement. The analog to digital converter (A/D) quantizes an analog signal in a finite amount of time. Speed of conversion is predicted in a finite amount of time by both the resolution of the converter and the frequency of the signal to be converted. Time required to perform a conversion is generally called the "aperature time".

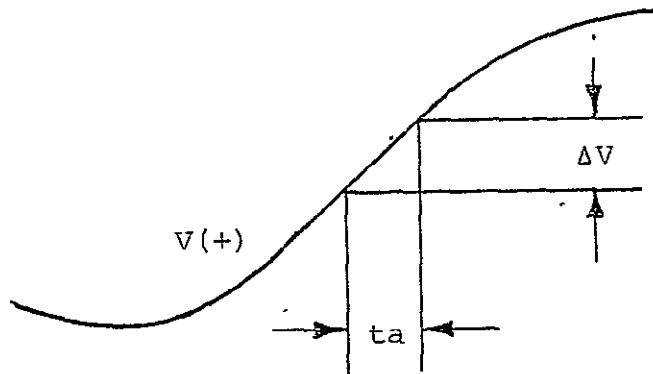


Fig. 4-8 APERATURE TIME AND AMPLITUDE UNCERTAINTY

As illustrated in Fig. 4-8, aperature time and amplitude uncertainty are related by the time rate of change of the analog signal. For the particular case of a sinusoidal signal to

be converted, the maximum rate of change occurs at the zero crossing of the waveform and the amplitude change is:

$$\Delta V = \frac{d}{dt} (V \sin wt)_{t=0} \times t_a \quad (4-1a)$$

$$\Delta V = V w t_a \quad (4-1b)$$

$$\text{giving } \frac{\Delta V}{V} = w t_a = 2 \pi f t_a. \quad (4-2)$$

From this result we can determine the aperture time required to digitize a 30 Hz signal to 12 bits resolution (a resolution of 1 part in 4096 or 0.0244%).

$$t_a = \frac{V}{\Delta V} \times \frac{1}{2 f} = \frac{.000244}{6.28 \times 30} = 1.3 \times 10^{-6}$$

This result indicates that to remain within 1 bit of resolution (0.0244%) we require an aperture time of 1.3 microseconds to process analog signals varying at a rate of 30 Hertz. It can be seen that the system would require fast A/D converters plus extremely fast computational capability to accommodate this configuration of sensors and analog subsystem. By using multiplexing and sample and hold circuitry we can however reduce the number of A/D converters required to one and alleviate the aperture and processing requirements imposed above.

The operation of sampling to be used by the instrument is illustrated in Fig. 4-9 which shows an analog signal and a train of sampling pulses. The pulses are provided by the central processing unit. A switch connects the analog signal for a very short period of time to the hold circuitry charging a capacitor and storing the sampled voltage until the next sample is required. This type of sampler is called sample and hold.

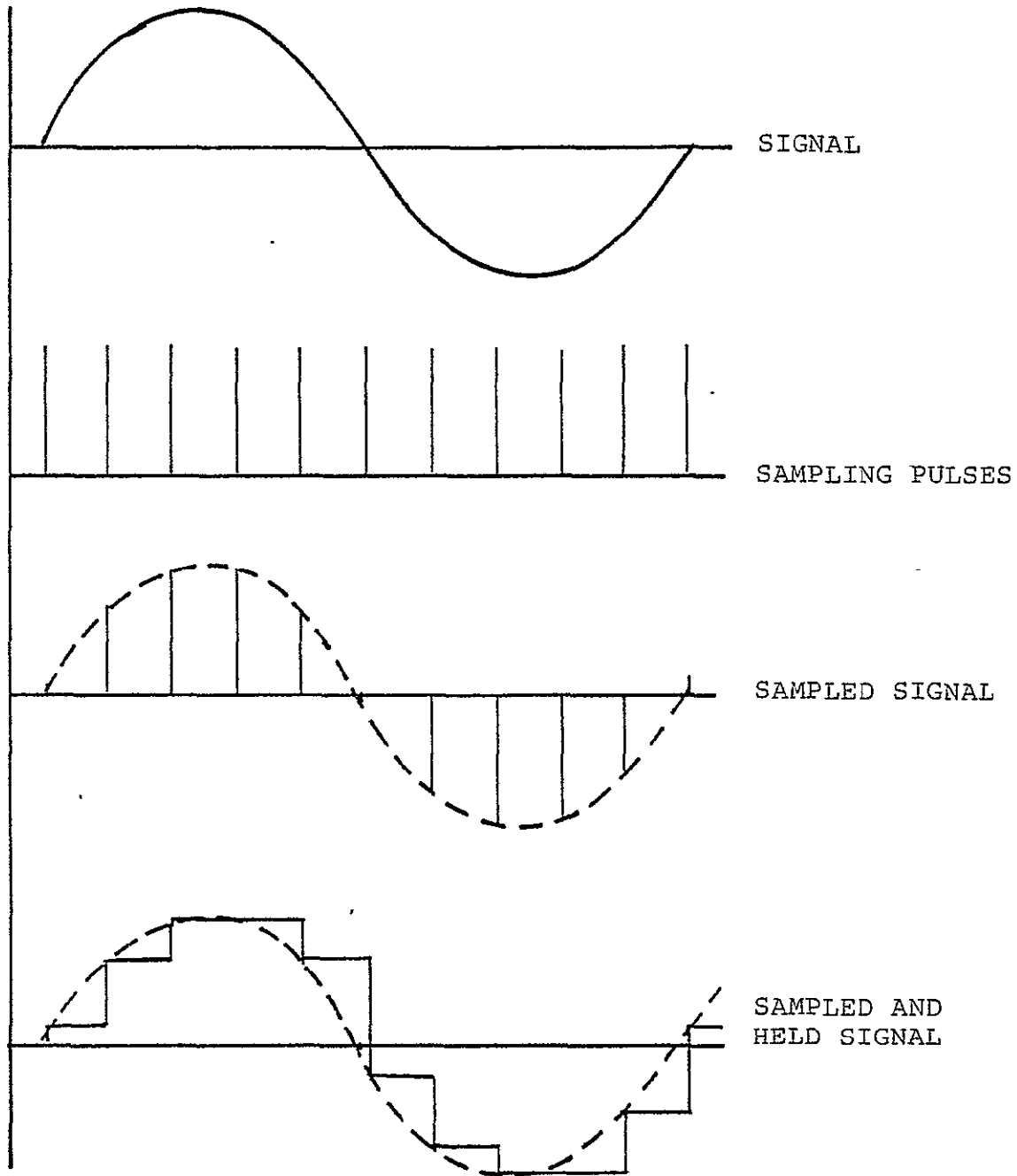


Fig. 4-9 SIGNAL SAMPLING PROCESS

### A) Sampling Rate Errors

The process of uniformly sampling a function of continuous time can yield a significant source of error if the sampling period  $T$  is selected too large [Ref. 4-4, 4-5]. This error can be illustrated by considering an analog signal  $x_a(t)$  that has the Fourier representation [Ref. 4-6]

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega \quad (4-3a)$$

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt \quad (4-3b)$$

The sequence  $x(n)$  with values  $x(n) = x_a(nT)$  is said to be derived from  $x_a(t)$  by periodic sampling and  $T$  is the sampling period. The reciprocal of  $T$  is called the sampling frequency or sampling rate. In order to determine the sense in which  $x(n)$  represents the original signal  $x_a(t)$ , it is convenient to relate  $X_a(j\Omega)$ , the continuous-time Fourier transform of  $x_a(t)$ , to  $X(e^{j\Omega})$ , the discrete-time Fourier transform of the sequence  $x(n)$ . From (4-3a) we note that

$$x(n) = x_a(nT) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega nT} d\Omega \quad (4-4)$$

From the discrete-time Fourier transform we also obtain the representation [Ref. 4-4]

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \quad (4-5)$$

To relate the equations (4-4) and (4-5) we can express (4-4) as a sum of integrals over intervals of length  $2\pi/T$ , as in

$$x(n) = \frac{1}{2\pi} \sum_{r=-\infty}^{\infty} \int_{(2r-1)\pi/T}^{(2r+1)\pi/T} X_a(j\Omega) e^{j\Omega nT} d\Omega \quad (4-6)$$

Each term in the sum can be reduced to an integral over the range  $-\pi/T$  to  $+\pi/T$  by a change of variables to obtain

$$x(n) = \frac{1}{2\pi} \sum_{r=-\infty}^{\infty} \int_{-\pi/T}^{\pi/T} X_a \left[ j \left( \Omega + \frac{2\pi r}{T} \right) \right] e^{j \left( \Omega + \frac{2\pi r}{T} \right) nT} d\Omega \quad (4-7a)$$

$$x(n) = \frac{1}{2\pi} \sum_{r=-\infty}^{\infty} \int_{-\pi/T}^{\pi/T} X_a \left( j\Omega + j\frac{2\pi r}{T} \right) e^{j\Omega nT} e^{j2\pi rn} d\Omega \quad (4-7b)$$

If we now change the order of integration and summation and note that  $e^{j2\pi rn} = 1$  for all integer values of  $r$  and  $n$ , we obtain

$$x(n) = \frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \left[ \sum_{r=-\infty}^{\infty} X_a \left( j\Omega + j\frac{2\pi r}{T} \right) \right] e^{j\Omega nT} d\Omega \quad (4-8)$$

By substituting  $\Omega = \omega/T$  we get

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[ \frac{1}{T} \sum_{r=-\infty}^{\infty} X_a \left( \frac{j\omega}{T} + j\frac{2\pi r}{T} \right) \right] e^{j\omega n} d\omega \quad (4-9)$$

which is identical in form to equation (4-5). We can therefore make the identification (equating like terms of (4-5) and (4-9))

$$X(e^{j\omega}) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X_a \left( \frac{j\omega}{T} + j\frac{2\pi r}{T} \right) \quad (4-10)$$

We can also express (4-10) in terms of the analog frequency variable  $\Omega$  (where  $\Omega = \omega/T$ ) as

$$X(e^{j\omega T}) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X_a \left( j\Omega + j\frac{2\pi r}{T} \right) \quad (4-11)$$

The last two equations clearly reveal the relationship between the continuous-time Fourier transform and the Fourier transform of a sequence derived by sampling. For example, if  $X_a(j\Omega)$  is as depicted in Fig. 4-10a then  $X(e^{j\omega})$  will be as shown in Fig. 4-10b when the sampling period  $T$  is too long and as shown in Fig. 4-10c if  $T$  is short enough.

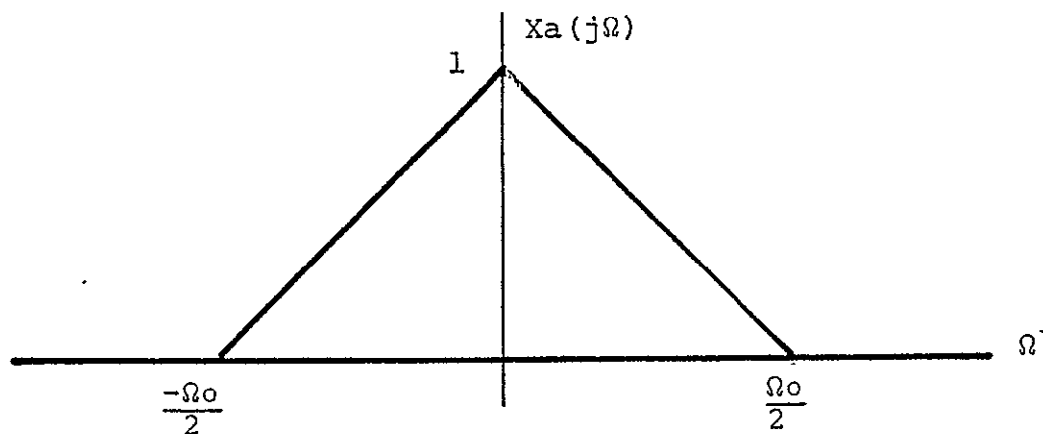
From Fig. 4-10c it is obvious that if  $\frac{\Omega_0 T}{2} < \pi$ , i.e., we sample at a rate at least twice the highest frequency of  $X_a(j\Omega)$ , then  $X(e^{j\omega})$  is identical to  $X_a(\omega/T)$  in the interval  $-\pi \leq \omega \leq \pi$  and can be recovered from the samples  $x_a(nT)$  by an appropriate interpolation formula.

For the remote magnetic indicator instrument designed in previous chapters, the analog signals are filtered with a low pass section reducing frequency content above 30 Hz. The sampling rate must therefore exceed 60 Hz ( $T < 16.67$  m.s.) to enable accurate dynamic operation of the system.

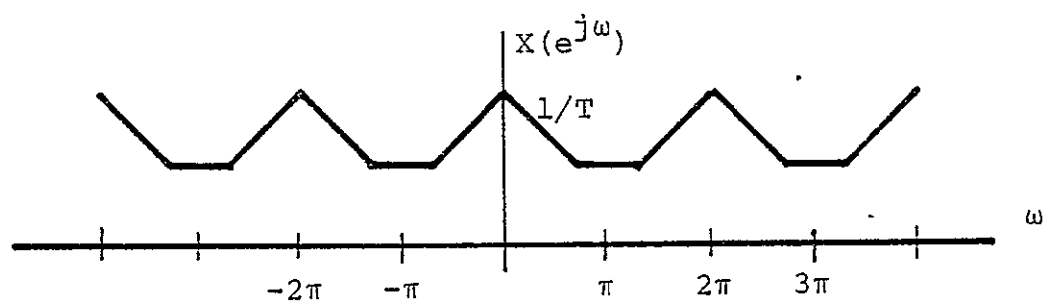
Laboratory measurements of sampling rates on the functional microprocessor based instrument revealed that the analog subsystem operated at a sampling rate of 62.5 Hz (16 m.s.) indicating that the algorithm execution rate supported a system bandwidth of 31.25 Hz. If frequency content of the analog signals is less than 31.25 Hz there is no error due to sampling.

#### B) Analog Multiplexer Induced Error

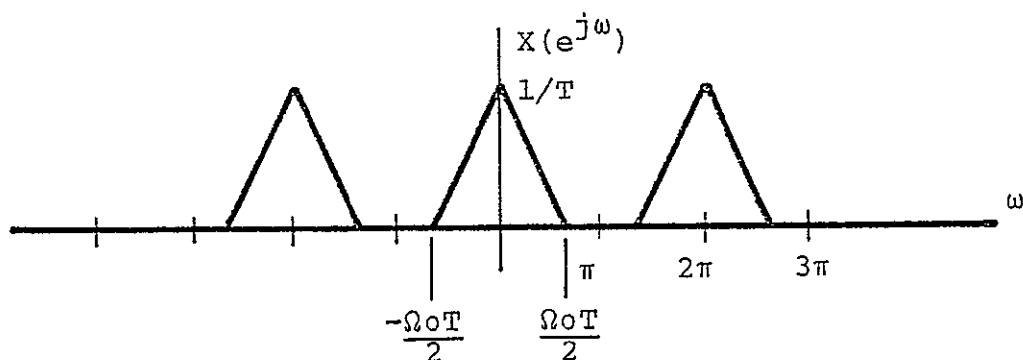
The analog multiplexer of Fig. 4-7 selectively connects one analog transducer output at a time to the input of the sample and hold subsystem. The Datel Systems, Inc., multiplexer [Ref. 4-7] selected for the remote magnetic indicator experiment features eight MOS-FET switches with associated driver circuits,



(a) Fourier transform as a continuous-time signal



(b) Fourier transform of the discrete-time signal obtained by periodic sampling ( $T$  is too large)



(c) Same as (b) except  $T$  is short enough

Fig. 4-10 FOURIER TRANSFORMS OF CONTINUOUS AND DISCRETE-TIME SIGNALS



FET pull-up to reduce propagation delays and all of the necessary decoding logic to enable random channel addressing with a four bit parallel binary input.

Several important parameters are used to characterize analog multiplexers and can contribute error.

#### 1) Transfer Accuracy

Transfer accuracy is a function of the source impedance, switch resistance, load impedance (if the multiplexer is not buffered) and the signal frequency. It expresses the input to output error as a percentage of the input. In our case the system configuration predicates a maximum error due to transfer accuracy of ( $\pm 0.01\%$ ) yielding an error term of

$$\pm 0.0001 \times 2.5 \text{ Volts} = \pm 25 \text{ mV}$$

#### 2) Settling Time

This parameter defines the time elapsed from the application of a full scale step input to the time when the output has entered and remained within a specified error band around its final value. In our case the selected multiplexer has a maximum settling time of 1 microsecond to  $\pm 0.01\%$  full scale (F.S.) Since the control system selecting channels is implemented using a microprocessor, the minimum time between analog subsystem commands will always be greater than 3.0 microsecond<sup>6</sup>. The multiplexer will therefore always have settled to the final value before the sample and hold circuit (following this subsystem) can be activated with no error due to the settling time parameter.

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<sup>6</sup>One machine cycle time for the 2650 microprocessor with 1 MHz clock frequency.

### 3) Throughput Rate

The highest rate at which the multiplexer can switch from channel to channel at its specified accuracy is in this case 500 kHz. Since this rate is more than four orders of magnitude greater than the operational rate of the subsystem there is no error due to throughput rate limitations.

### 4) Input Leakage Current

The amount of signal coupled to the output as a percentage of input signal applied to all OFF channels together can be calculated by considering the maximum leakage current specified from OFF channels to the ON channel. In our case the maximum error signal can be calculated

$$\text{Error} = [ 4 (8 \text{ na} \times 2000 \text{ ohms source imped.})^2 ]^{\frac{1}{2}}$$

$$\text{Error} = 32 \text{ microvolts}$$

Note that in this case the voltage levels are statistically independent allowing an R.S.S. of error sources to calculate total error [Ref. 4-8, 4-9].

### C) Sample and Hold Circuit Induced Errors

The sample and hold subsystem consists of a switch and capacitor arrangement as shown in Fig. 4-11. The Datel Systems, Inc., model SHM-IC-1 integrated circuit sample and hold device [Ref. 4-10] features a self-contained high gain differential input amplifier, a digitally controlled electronic switch and a high input impedance buffer amplifier. The external components used with the sample and hold circuit in the solid state remote magnetic indicator instrument consisted of the 0.001 $\mu$ f holding capacitor and a 100K offset trimpot. By connecting

the output back to the negative input of the input amplifier (Fig. 4-11), the sample and hold subsystem operated in a unity gain, noninverting mode. When the switch is closed, the unit is in the sampling or tracking mode (Digital Control = 0 Volts), and will follow a changing input signal.

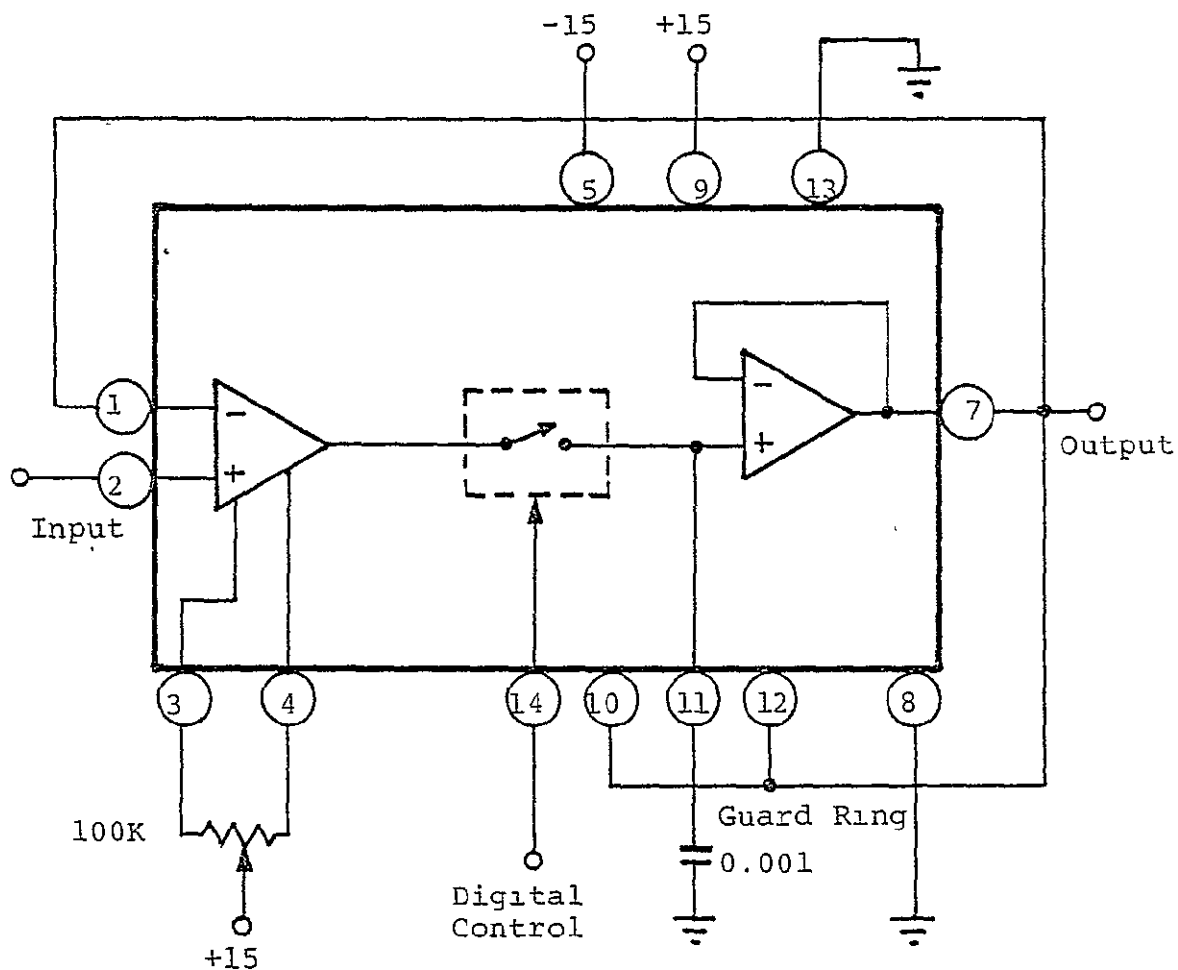


Figure 4-11 SAMPLE AND HOLD SUBSYSTEM

When the switch opens the unit is in the hold mode and retains a voltage on the capacitor for some period of time depending on capacitor and switch leakage. Sample and hold devices are characterized by a number of important parameters that must be considered in the design of a data acquisition subsystem.

### 1) Acquisition Time

The time lapse between the time that the sample command is given to the point where the output enters and remains within a specified error band around the input value is specified to be less than 4 microseconds time to transit from 0 to 0.1% of 10 Volts with  $C = 0.001 \mu\text{f}$  [Ref. 4-10]. This implies that the control signals emanating from the central processor should allow at least 4  $\mu\text{s}$  acquisition time prior to entering the hold mode. We note that the sample and hold subroutine (Appendix B) executes the instruction

IORI, R3 H'80' READY TO HOLD,

a two machine cycle instruction prior to sending the hold control signal. This instruction delays control signal transmission by  $(2 \times 3 \mu\text{s}) = 6 \mu\text{s}$  allowing the sample and hold circuit ample time to settle with no appreciable error due to the acquisition time parameter.

### 2) Hold Mode Voltage Droop

The maximum change in output voltage as a function of time is specified to be 50 mv/sec maximum using a  $0.001 \mu\text{f}$  polystyrene capacitor. Since the maximum total accumulated time to completion of the analog to digital conversion can be calculated as

5 Instructions (11 machine cycles)	= 33 $\mu$ s
1 Analog to Digital Conversion	= 20 $\mu$ s
3 Instructions if Conversion not synchronized with instructions (7 machine cycles)	= <u>21</u> $\mu$ s
	74 $\mu$ s

we can then compute droop error to be  $50 \text{ mv/sec} \times (74 \times 10^{-6})$   
 $\text{sec} = 3.73 \text{ mv.}$

### 3) Aperature Delay

The maximum time lapse between the time of hold signal receipt to opening of the switch is specified to be 50 nsec, an insignificant length of time in the instrument. There is therefore no error due to aperature delay.

### 4) Offset Error

Although the maximum offset error is specified to be 20 mv maximum [Ref. 4-10], the error was eliminated using the 100K trimpot offset adjustment. There was no appreciable offset error contribution due to the sample and hold circuit.

### 5) Gain Error

The gain error of a sample and hold circuit is apparent during the sample mode when the transfer function of the total amplifier deviates from the ideal unity slope condition (Fig. 4-12). In the noninverting unity gain mode, the specified gain error is  $\pm 0.05\%$  maximum yielding a signal error of

$$(\pm 0.0005) \times 5.0 \text{ V} = 250 \text{ mV}$$

This error can, however, be eliminated with the gain

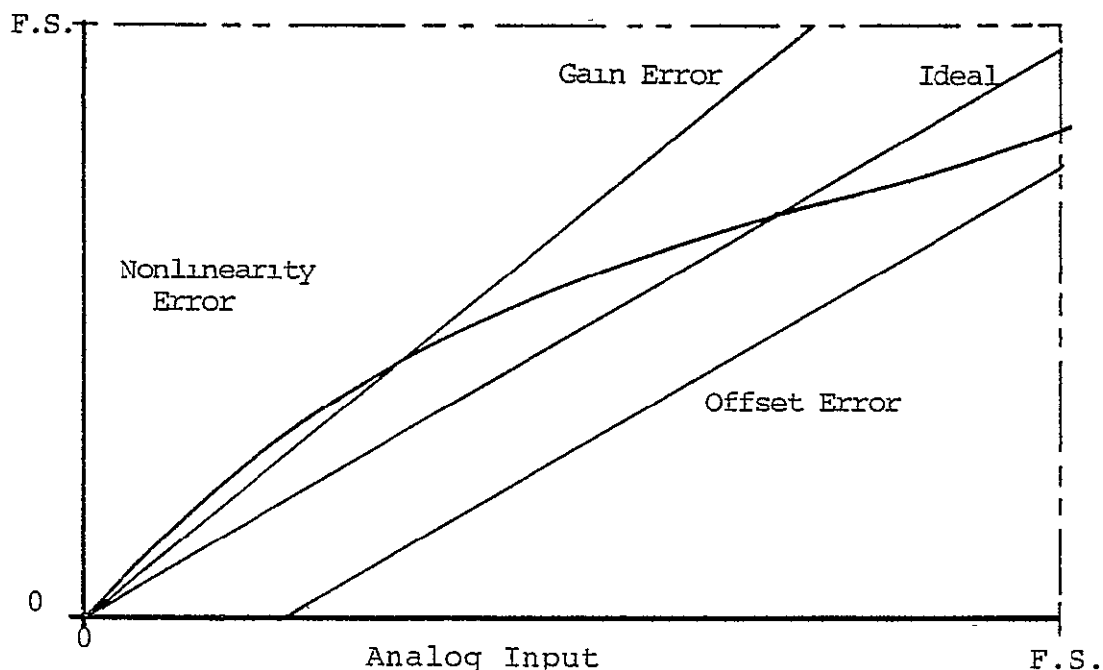


Fig. 4-12 GAIN, OFFSET AND LINEARITY ERRORS

adjustment available at the analog to digital converter. There will therefore be no appreciable net gain error due to the analog subsystem.

#### 6) Nonlinearity Error

Nonlinearity error is apparent in the sample and hold circuit if the transfer function departs from a linear curve (Fig. 4-12). In the noninverting unity gain mode with a 0.001  $\mu$ f holding capacitor the maximum nonlinearity is 0.01% resulting in a worst case signal uncertainty of  $(.0001) \times 2.5 \text{ Volts} = 25 \text{ mV}$ .

#### 7) Hold Mode Feedthrough

This error appears due to input signal appearing at the output when the unit is in the hold mode. Although the

feedthrough varies with signal frequency and the expected signal frequencies are substantially lower than the upper frequency limits of the sample and hold device (30 Hz max. versus several kiloHertz), we consider the worst case feedthrough of 0.01% [Ref. 4-10] or 25 mV.

#### D) Analog to Digital Converter Induced Errors

The A/D Converter selected for the solid state magnetic indicator instrument (Datel ADC-MA12B1B) [Ref. 4-11] uses the successive approximation technique to achieve excellent linearity and speed. Important parameters that potentially contribute errors are addressed below.

##### 1) Resolution Error

The smallest analog change that can be distinguished by the A/D converter is

$$\text{Least Significant Bit (LSB)} = \frac{\text{Full Scale}}{2^n}$$

$$\text{LSB} = \frac{5}{2^{12}} = 1.22 \text{ mV}$$

this uncertainty manifests itself as an error in computing by limiting the precision of any calculation.

##### 2) Linearity Error

The maximum deviation from a straight line drawn between the end points of the converter transfer function are specified in [Ref. 4-11] to be  $\pm 1/2$  LSB (in our case  $\pm 1.22$  mV of analog signal).

C-2

### 3) Accuracy Error

The input to output error of the A/D converter is specified in [Ref. 4-11] to be  $\pm 0.012\%$  F.S.  $\pm 1/2$  LSB or

$$\pm(0.00012) \times 5.00V \pm 1.22 \text{ mV} = \pm 1.82 \text{ mV Worst Case}$$

In reality, the two error terms are unrelated and the

$$\text{Rss Error} = \pm \left[ (.00012 \times 5V)^2 + (1.22\text{mV})^2 \right]^{\frac{1}{2}}$$

$$\text{RSS Error} = \pm 1.36 \text{ mV}$$

### 4) Offset Error and Gain Error

Both the offset error and gain error were adjusted to zero using the trimming potentiometers (Fig. 4-7) and the calibration procedure outlined in Ref. 4-11. A reference signal of plus  $1/2$  LSB (1.22 mV) was applied to the converter and the offset trimming potentiometer adjusted until the output flickered equally between logic "0" and logic "1". The gain was then adjusted by setting the converter input to full scale minus  $1-1/2$  LSB (4.99817 Volts) and the gain trimming potentiometer was adjusted until the output flickered between logic "111...110" and logic "111...111". The above steps were repeated until no appreciable error in gain or offset was evident.

## 4-4 PROCESSING ERRORS

Errors in processing data accrue due to several sources including imprecision and truncation. Since the microprocessor selected for the instrument is inherently an eight bit device, single precision calculations are conducted with eight bits and double precision calculations are conducted with a total



of sixteen bits. This section addresses the effects of computational precision and truncation in the various subroutines and relates these to overall computational accuracy. The various subroutines are analyzed in chronological order as they appear in the main program.

A) Subroutine "SAMP"

The sample subroutine (delineated in Fig. 3-6a) selects and digitizes analog signals by controlling respective analog subsystem modules. During the first portion of this subroutine, A/D converter data bits are stored in two consecutive bytes<sup>7</sup> in the computer memory. The A/D conversion precision of 12 bits is thereby preserved.

The second, third and fourth operations of the sample subroutine convert the unipolar binary format of the data to sign magnitude format, adds offset quantities and merely changes the signs of the Hx and Hy data. The operations are conducted in a double precision manner and precision of the data remains unaltered.

Correction of x axis orthogonality error is the final operation of the sample subroutine. Equation (3-1) is implemented at this point using a table lookup (for the sin function), multiplication and addition. The final result can be expressed as

$$Hx = Hx' + Hy \sin \epsilon$$

---

<sup>7</sup>A byte is accepted terminology for an eight bit data quantity.

where the respective quantities have the following forms

$$\begin{aligned} Hx^1 &= x_1 + \sum_{i=1}^{11} a_i 2^i \\ Hy &= x_2 + \sum_{j=1}^{11} a_j 2^j \\ \sin \epsilon &= \sum_{k=1}^8 a_k 2^{-k} \end{aligned}$$

and

$x_1, x_2$  are sign bits

$a_{i,j,k}$  equal 0 or 1 depending on whether the respective term is to exist or not

We can analyze the effects of imprecision and truncation by noting that the sin function has eight significant binary bits resulting in a resolution of  $1/256$  or  $90^\circ/256 = 0.352^\circ$ .

The relative error in  $\sin \epsilon$  is computed by Dahlquist [Ref. 4-12] as follows

let  $\tilde{a}$  = the approximate value of  $\sin \epsilon$   
 $a$  = the exact value of  $\sin \epsilon$

then the relative error in  $\tilde{a}$  is

$$(\tilde{a} - a)/a \text{ if } a \neq 0$$

Since data in the sin table has been truncated, maximum relative error can be as large as  $\pm(1/2^{12})$  or  $\pm 0.02\%$ .

From the definition of relative error we obtain the following relationships between exact, estimate and estimated

relative error

$$\tilde{a} = a + ar = a(1 + r)$$

If  $a_1$ , and  $a_2$  have relative errors of  $\pm 0.39\%$  and  $\pm 0.02\%$ , respectively, then

$$\begin{aligned}\tilde{a}_1 \tilde{a}_2 &= a_1(1 \pm 0.0039) a_2(1 \pm 0.00024) \\ &= a_1 a_2 (1 \pm 0.0039)(1 \pm 0.00024)\end{aligned}$$

Thus, the relative error in  $\tilde{a}_1 \tilde{a}_2$  is

$$\begin{aligned}(1 \pm 0.0039)(1 \pm 0.00024) - 1 &= \\ \pm(0.0039) \pm(0.0039)(0.00024) \pm(0.00024) &= \\ \approx \pm(0.0041)\end{aligned}$$

Since the maximum value of  $\sin \epsilon$  to be encountered occurs when the orthogonality error ( $\epsilon$ ) is 1 degree,  $\sin \epsilon = 0.017$  maximum. The maximum value for  $H_y$  can be 0.6 gauss or 2048 units. Maximum error due to imprecision in the product  $H_y \sin \epsilon$  is then

$$\begin{aligned}\text{Er Max} &= (2048 \times 0.017)(1 \pm 0.0041) - (2048 \times 0.017) \\ &= 0.1427 \text{ units}\end{aligned}$$

Since only the integer portion is retained in the final product, insignificant error can be attributed to imprecision of the  $\sin \epsilon$  term in this case. Orthogonality error will be adequately corrected.

#### B) Subroutines ROTX and ROTY

These subroutines were developed in Chapter III and implement the equation of 2-11 required to compute horizontal

x and y magnetic field components. Equations to be implemented by the respective subroutines are

$$\begin{aligned} H_x = & H_{xm} \cos(\text{pitch}) + H_{ym} \sin(\text{pitch}) \sin(\text{roll}) \\ & + H_{zm} \sin(\text{pitch}) \cos(\text{roll}) \end{aligned} \quad (4-12)$$

and

$$H_y = H_{ym} \cos(\text{roll}) - H_{zm} \sin(\text{roll}) \quad (4-13)$$

where  $H_{xm}$ ,  $H_{ym}$  and  $H_{zm}$  are measured field components made available from the magnetometer via the analog subsystem.

Since the transcendental functions are implemented using table lookup and are limited in precision to 8 bits, imprecision in these variables will dominate in generating error. In particular, the sin/cos terms will have relative error in the order of  $\pm 1/256$  or  $\pm 0.39\%$  while the measured field data has relative uncertainty of only  $\pm 1/4096$  or  $\pm 0.02\%$ . Multiplications will result in addition of the bounds for the relative error as illustrated in section 4-4A above.

The transcendental terms above are limited in magnitude to 1.0 maximum while the field measurements can be 0.60 gauss max. In this case the individual product terms of (4-12) and (4-13) can have maximum errors of

$$E_r = (2048)(1 + 0.0041) - (2048) = 8.4 \text{ units}$$

Errors in  $H_y$  and  $H_x$  (4-12 and 4-13) will be maximum when roll and pitch are at 45 degrees and the fields are equal. In this case the error in  $H_y$  will be

$$\begin{aligned} EHy &= [(0.707)(2048)(1 + 0.0041) - (0.707)(2048)] - \\ &\quad [(0.707)(2048)(1 - 0.0041) - (0.707)(2048)] \end{aligned}$$

$$EHy = 4.94 - 5.94 = 11.87 \text{ units}$$

Similarly, maximum error in Hx can be calculated as

$$\begin{aligned} EHx &\approx [(0.707)(2048)(1.0041) - (0.707)(2048)] \times 3 \\ EHx &\approx 17.8 \text{ units maximum} \end{aligned}$$

It should be noted that these error terms are worst case and peak at multiples of 45 degrees in yaw.

#### C) Subroutines COSY and SINY

These two subroutines compute the angle between the x axis sensor (when projected onto the horizontal plane) and the north-south horizontal vector of earth's magnetic field. The first two operations of these subroutines perform double precision multiplication and division. Since the data variables involved are 12 bits in length and the computations performed preserving 16 bits, no error is introduced.

The "ANGL" subroutine called by the above two subroutines computes the desired (x axis to horizontal vector) angle by completing an associative table look up procedure. The task required is to match a given data quantity either  $(Hx^2/Hh^2)$  or  $(Hy^2/Hh^2)$  with the contents of a memory cell. The address of this cell is then the required angle.

Since the table is limited in precision to 16 bits there are obviously cases where an interpolation is required to

ascertain the true address<sup>9</sup>. The function stored in tabular form is  $\cos^2\theta$  where  $\theta$  varies from 45 to 90 degrees. Maximum error will therefore be induced while attempting to locate solutions (angles near 90 degrees if inadequate precision is provided. Error in this region due to resolution of tabular data can be examined by noting the entries in Table 4-8

$\theta$	$\cos^2\theta$	Most Significant Binary Bit ( $2^{-x}$ )
90	0	-
89	0.000305	12
88	0.001218	9
87	0.00274	8

Table 4-8  $\cos^2\theta$  AND MOST SIGNIFICANT BINARY DIGITS

provided to indicate the relative magnitudes of  $\cos^2\theta$  in the region of  $\theta = 90$  degrees. We observe that the most significant binary digit affected at 89 degrees is binary decimal digit 12 implying that the resolution of  $H_x^2/H_h^2$  or  $H_y^2/H_h^2$  (the argument of  $\cos^2\theta$ ) must be accurate to at least  $1/2^{12}$  or 0.024%.

Considering the horizontal field of earth's magnetic vector as observed in laboratory experimentation at this latitude, we note that  $H_h$  is 730 units. At a heading of 89 degrees,  $H_x = 730 \cos 89 = 12.7$  units. The argument would therefore be

$$\text{ARG} = H_x^2/H_h^2 = \frac{(12.7)^2}{(730)^2} = 0.000305$$

<sup>9</sup>The procedure determines the relative address by linear interpolation, then selects the closest address as the required angle for the solution.

Since, the squaring and division operations are conducted in double precision, precision is preserved and the algorithm should be able to resolve heading to at least one degree over all portions of the compass.

#### D) Errors Due to the Remaining Subroutines

Since all of the remaining subroutines work with data that has been rounded to a precision representing 1 degree or better and the computations involve addition or subtraction in double precision binary or binary coded decimal (BCD) format, we note that there will be no further appreciable error due to truncation or rounding.

### 4-5 MEASUREMENT ERROR SUMMARY

Errors due to sensors and measurement of their respective outputs were discussed in sections 4-2 and 4-3 above. Before proceeding with the analysis of errors, the total signal inaccuracy due to contribution from the many sources above will be summarized in Table 4-10. Total instrument error can then be computed by considering the propagation and enhancement of these errors during the computation process.

Since the errors in Table 4-9 are stochastically independent, we can compute error for any given signal level by finding the RSS of respective error sources. In this manner, the instrument error can be evaluated by considering all input signals with errors superimposed to produce an erroneous computation of heading.

PARAMETER	ERROR	COMMENT
1. Magnetometer		
Offset	$\approx 0$	Corrected by software
Orthogonality	$\approx 0$	Corrected by software
Noise	Negligible	
Gain	$\pm 0.01\%$	Proport. to signal level
Linearity	$\pm 0.01\%$	
2. Analog Subsystem		
Sampling	Negligible	Sampling rate & filtering adequate
Multiplexer		
Transfer Accuracy	$\pm 0.01\%$	Proport. to signal level
Settling Time	$\approx 0$	
Rate	$\approx 0$	
Input Leakage	$\approx 0$	
Sample and Hold		
Acquisition	$\approx 0$	
Hold	4mV	
Aperature Delay	$\approx 0$	
Offset	$\approx 0$	Corrected by software
Gain	$\approx 0$	Corrected by software
Nonlinearity	$\pm 0.01\%$	Proport. to signal level
Feedthrough	$\pm 0.01\%$	Proport. to signal level and frequency
A/D Converter		
Resolution	+1.2mV	
Accuracy	$\pm 1.4\text{mV}$	
Offset	$\approx 0$	
Gain	$\approx 0$	

Table 4-9 SENSOR AND ANALOG SUBSYSTEM ERROR SUMMARY



## 4-6 SAMPLE ERROR ANALYSIS

Orthogonality correction using the algorithmic method can be verified by computing expected error prior to correction and comparing measured system output with the error prediction. Assuming that the angle between the x and y sensors exceeds 90 degrees as in Fig. 4-13, we can proceed to compute error by noting the following relationships

$$\begin{aligned} H_x &= H_h \cos(+\psi) \\ H_y &= H_h \sin(\psi) \\ H_{x_1} &= H_x \cos \epsilon - H_y \sin \epsilon \\ \text{True Yaw} = \psi_T &= \cos^{-1} \left( \frac{H_x}{(H_x^2 + H_y^2)^{\frac{1}{2}}} \right) \end{aligned}$$

Computed yaw

$$\begin{aligned} \psi_m &= \cos^{-1} \left[ \frac{H_x \cos \epsilon - H_y \sin \epsilon}{(H_x \cos \epsilon - H_y \sin \epsilon)^2 + H_y^2}^{\frac{1}{2}} \right] \\ &= \cos^{-1} \left[ \frac{H_h \cos \psi \cos \epsilon - H_h \sin \psi \sin \epsilon}{[(H_h \cos \psi \cos \epsilon - H_h \sin \psi \sin \epsilon)^2 + H^2 h \sin^2 \psi]^{\frac{1}{2}}} \right] \end{aligned}$$

Using small angle approximations with  $\epsilon \approx 0.79^\circ$

$$\cos \epsilon \approx 1 \text{ and } \sin \epsilon \approx 0.014$$

then

$$\psi_m = \cos^{-1} \left[ \frac{H_h \cos \psi - 0.014 \sin \psi}{[H_h^2 (\cos \psi - 0.014 \sin \psi)^2 + H_h^2 \sin^2 \psi]^{\frac{1}{2}}} \right]$$

Computed error

$$\text{Error} = \psi_m - \psi$$

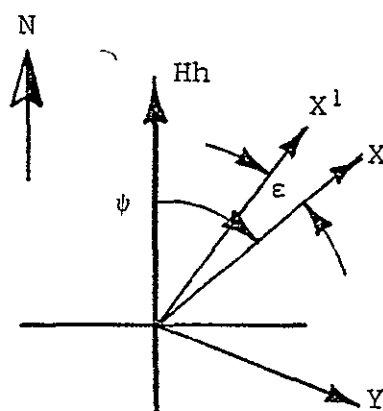


Fig. 4-13 ANGLE (X - Y) &gt; 90°

We can now evaluate computed yaw angle ( $\psi_m$ ) given a particular yaw ( $\psi$ ) and the horizontal field vector (Hh). Heading error for horizontal field vector of 730 units at various yaw angles with pitch and roll angles of zero degrees is tabulated in Table 4-10 and plotted along with actual measured yaw error (data taken during experimentation of Chapter V) in Fig. 4-14.

Heading (Degrees)	Computed Error (Degrees)	Heading (Degrees)	Computed Error (Degrees)
90	0.8	290	0.7
70	0.7	270	0.8
50	0.5	250	0.7
30	0.2	230	0.5
10	0.0	210	0.2
350	0.0	190	0.0
330	0.2	170	0.0
310	0.5	150	0.2
		130	0.5

Table 4-10 COMPUTED HEADING ERROR WITH Hh = 730 UNITS

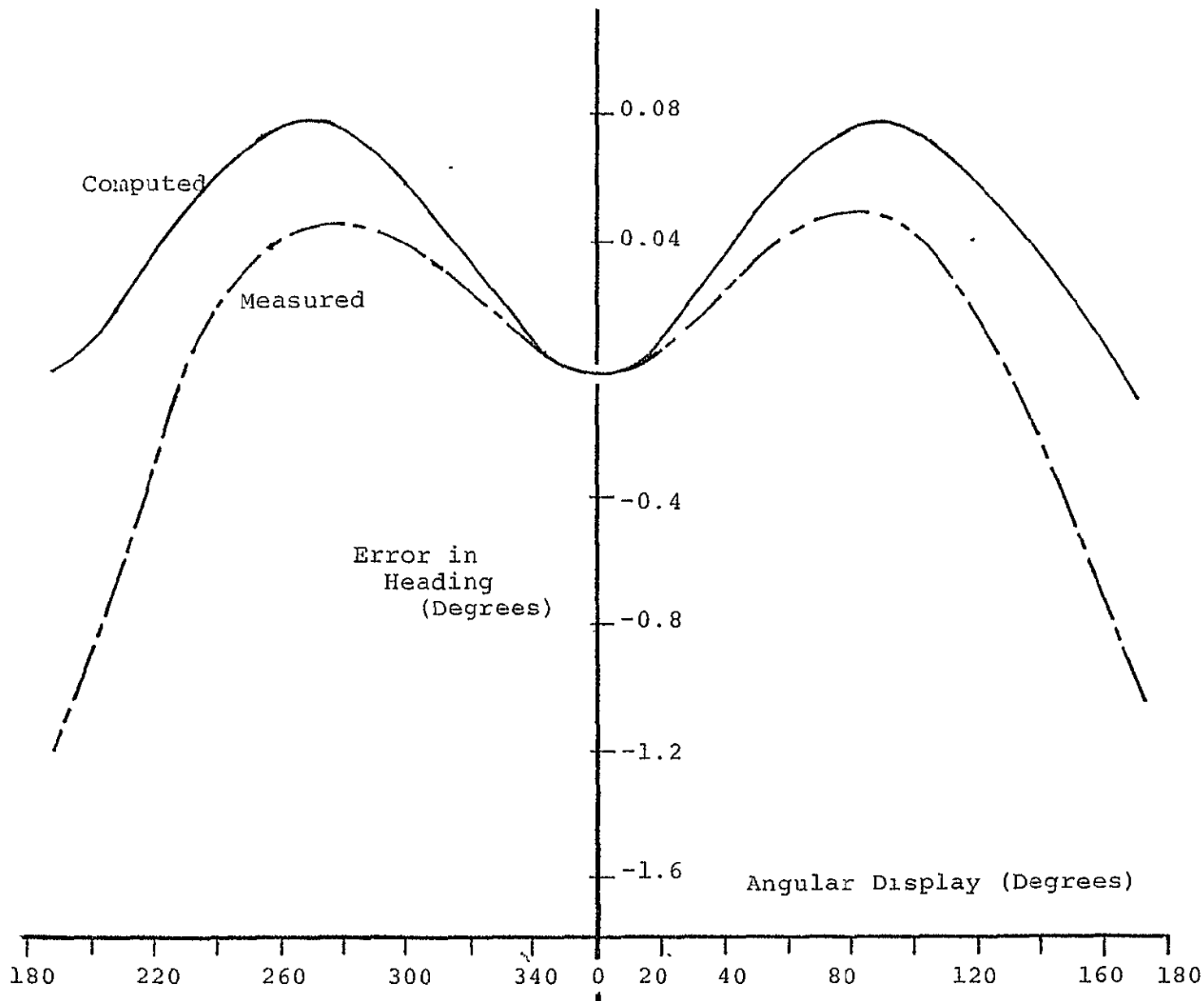


Fig. 4-14 COMPARISON OF COMPUTED AND MEASURED ORTHOGONALITY INDUCED ERROR

#### 4-7 CONCLUSIONS

The preceeding error analysis has identified potential error sources along with relative magnitudes of error to be expected. Magnetometer sensor and analog subsystem errors were identified and analyzed individually. During this analysis it became apparent that errors due to sensor offset and nonorthogonality dominated and would severely limit total instrument performance. The relative magnitudes of these errors and their mode of contribution would have degraded system capacity.

By carefully characterizing the offset and orthogonality error it was determined that these systematic errors could be reduced by appropriate programming. A need to identify the extent of each error unique to the laboratory instrument imposed a need to evaluate the instrument empirically. Using earth's magnetic field and the laboratory test fixture (described in Chapter V) to provide control inputs each of the parameters was identified and measured. An algorithm with the empirically determined correction coefficients was included in the final system to reduce the error and to improve final system performance. The remaining potential error sources were tabulated and relative magnitudes noted.

Processing errors due to register precision and truncation were analyzed by considering pertinent subroutines individually. It was noted that the relative error bounds add when multiplying variables with relative error. In addition, it was noted that error accrued during processing is proportional to sensor signal levels involved. The final uncertainty is then proportional to actual aircraft attitude with error increasing as displacement from level flight occurs. Computational error is also noted to increase at particular headings causing the error function to peak at specific yaw angles.

The sample error analysis clearly shows that a correlation between sensor nonorthogonality induced error and measured (uncorrected) data exists. By predicting and computing an error function prior to experimentally verifying the result we gain confidence that the sensor characteristics derived empirically in previous sections are correct.

## CHAPTER V

LABORATORY EVALUATION OF THE ATTITUDE INDEPENDENT  
REMOTE MAGNETIC INDICATOR AND HEADING INSTRUMENT

## 5-1 INTRODUCTION

This chapter addresses laboratory evaluation of the microprocessor based computer designed to implement the heading measurement instrument. An integral part of this instrument was the three axis fluxgate magnetometer used to implement the attitude independent remote magnetic indicator of Chapter II. The laboratory evaluation was designed to investigate empirically the effects of physical parameters that would otherwise be impossible to assess.

Although phenomena such as noise, magnetic field gradient, sensor orthogonality errors and offset errors can be predicated, combined effects on the proposed instrument and remote magnetic indicator are best evaluated in the laboratory. In addition, it was noted that errors due to sensor offset and nonorthogonality could be corrected by software included with the sample subroutine. Determination of the effectiveness of this correction technique necessitated laboratory measurements of the errors (to determine correction constants) and comparison of data prior to and following corrections.

The chapter begins by discussing laboratory test apparatus designed to evaluate the instrument. Actual data measured and recorded during experimentation is then presented in both tabular and graphic form to facilitate comparison and evaluation. Finally, the laboratory data is discussed and it is concluded that the remote magnetic indicator used with the heading measurement instrument results in a viable alternative to conventional heading measurement systems. The microprocessor based

computer implementation of the instrument has added unique sensor measurement correction ability that enhances performance of otherwise marginal sensors. In this manner limitations in systems performance that now exist due to sensor inadequacy can be minimized without incurring the burden of using more expensive sensors.

## 5-2 TEST APPARATUS

### A) Electronic Subsystem

The microprocessor based computer (illustrated in photos 5-1 and 5-2) was constructed on printed circuit boards consisting of a central processing card, two memory cards (2K bytes capacity each) and an output board. A separate analog subsystem card contained the multiplexer, sample and hold, analog to digital converter and trimming potentiometers. The circuit cards were all organized with edge connectors and mounted vertically into a hand wired backplane assembly as shown in photos 5-1 and 5-2.

The card in the left foreground of photo 5-1 served as the output display with three seven-segment displays displaying significant figures of system heading. A small printed circuit in the right foreground of photo 5-1 contained potentiometers used to generate analog signals proportional to roll and pitch signals (simulating gyroscope outputs). Cards shown vertically mounted in photo 5-2 can be identified from right to left as the analog subsystem, two memory cards and the central processing card. The large integrated circuit shown on the CPU card is the Signetics 2640 microprocessor.

### B) Sensor Assembly

To evaluate the effects of combined aircraft pitch, roll



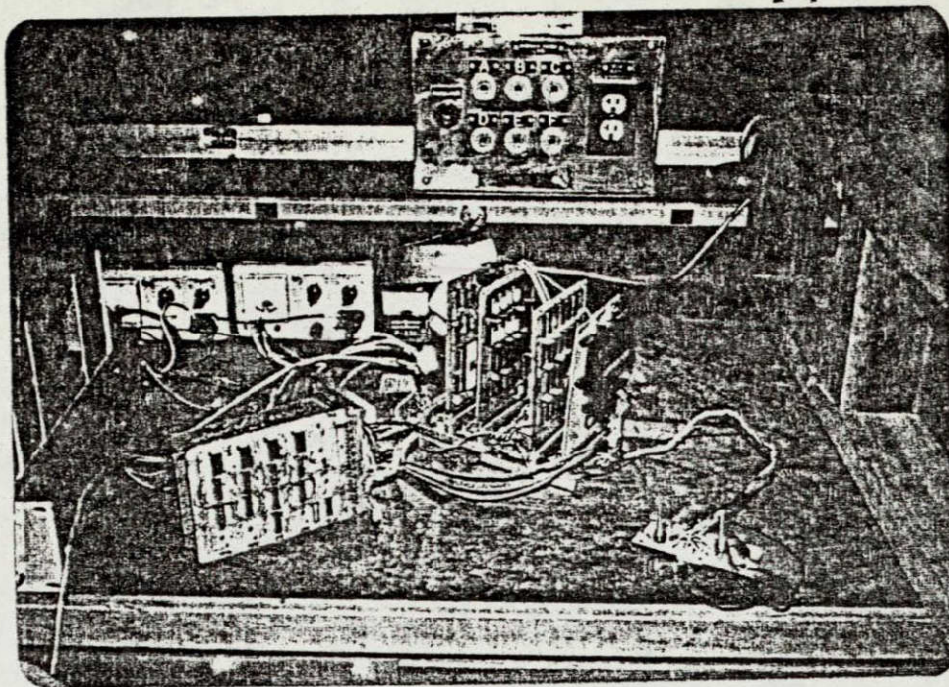


Photo 5-1 . MICROPROCESSOR BASED HEADING COMPUTER

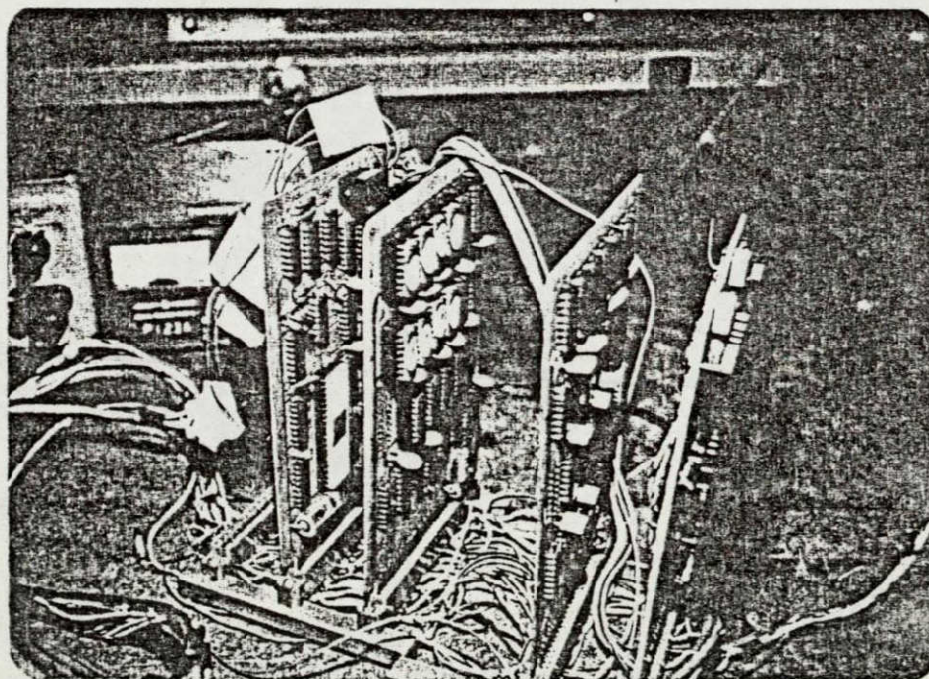


Photo 5-2 CENTRAL PROCESSOR, MEMORY AND  
ANALOG SUBSYSTEM



and yaw a three axis gimbal apparatus was required. In addition, since angular measurements were required, a means of measuring angular rotation in each of the three axes was provided. The gimbal apparatus as illustrated in photos 5-3 and 5-4 was fitted with large protractors centered on the rotation axes. Pointers were provided to enable angular rotation measurements on the respective protractor scales. Since the angular precision on each protractor scale resolved angular position to 0.5 degrees, angular measurements to a resolution of at least 0.5 degrees were possible. Angular position was measured by estimating the decimal place of each measurement with accuracy to  $\pm 0.5$  degrees ensured.

Since the three axis magnetometer (housed in the rectangular block of photos 5-3 and 5-4) measured ambient magnetic fields the test apparatus was constructed of nonferrous material. This ensured that local fields due to residual magnetic fields in the test apparatus would be minimized. In addition, since the material had low permeability, there would be little deformation of the local field causing error due to changing field gradient.

The sensor package shown in photos 5-3 and 5-4 was physically mounted such that the sensors were centered as close to the center of the gimbal as possible. This precaution ensured that measurement error due to sensor translation was minimized<sup>1</sup>. During instrument evaluation, the entire gimbal assembly and sensor were leveled and mounted in a Helmholtz coil assembly as illustrated in photo 5-5. Although the coils were not activated during experimentation, the rotations in heading were

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<sup>1</sup>Since the local magnetic field has a nonzero gradient, field measurements include a component due to translation of the sensor axes. This component of measurement produces unacceptable error in a system designed to measure field components that change due to rotation.



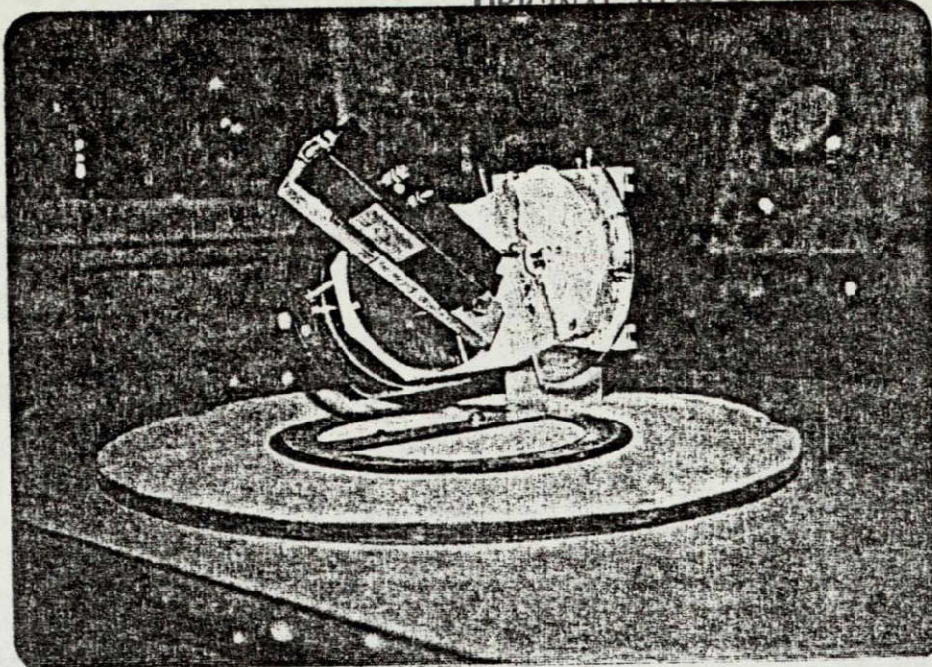
REPRODUCIBILITY OF THE  
ORIGINAL DESIGN

Photo 5-3 MAGNETOMETER SENSOR MOUNTED ON  
GIMBALLED TEST FIXTURE

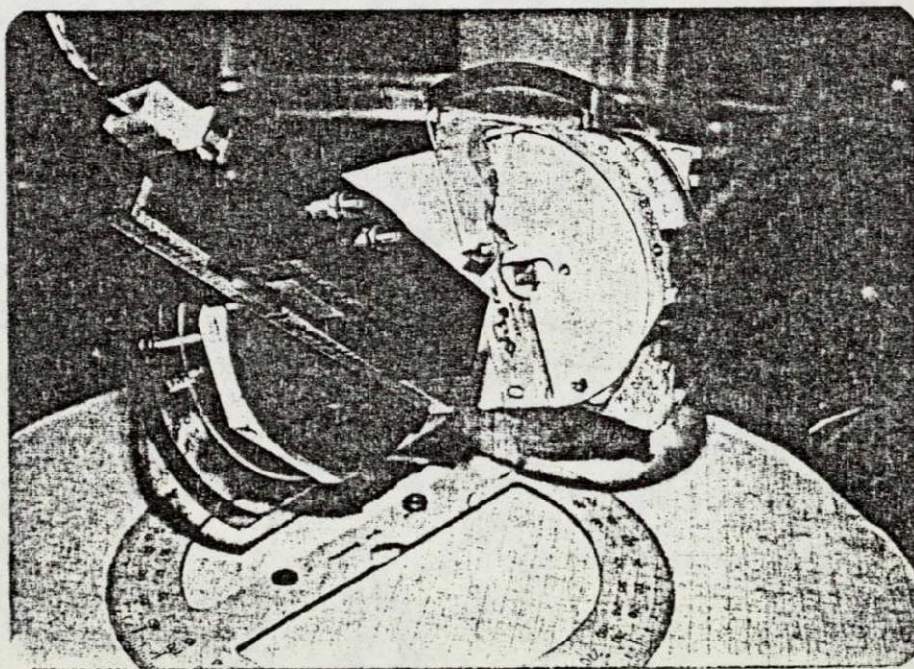


Photo 5-4 SENSOR AND GIMBAL ASSEMBLY WITH PROTRACTORS



carefully controlled since the gimbal assembly was an integral part of the Helmholtz coil fixture with the vertical rotation axis serving as the system yaw axis.

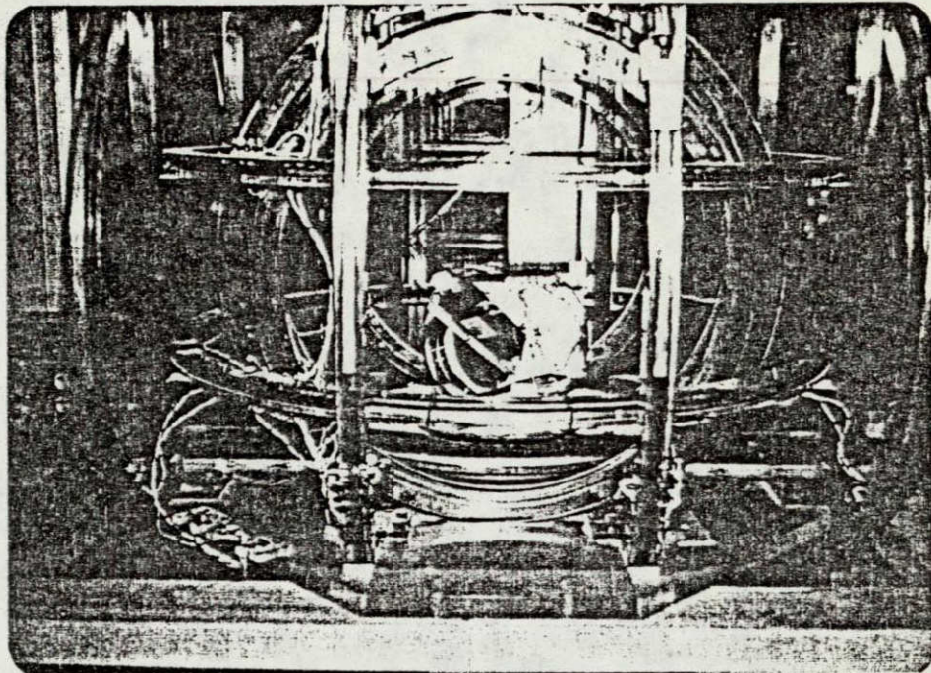


Photo 5-5 TEST FIXTURE MOUNTED IN HELMHOLTZ COIL ASSEMBLY

### 5-3 HEADING MEASUREMENTS WITH NO OFFSET CORRECTION

By maintaining heading of the test fixture constant (no rotation about the vertical axis) and varying both pitch and roll angle, the instrument display was observed to vary. This variation gave a direct measure of instrument error since a constant heading was maintained and a constant display was to be expected.

Data variations were recorded in Tables 5-1 and 5-2 and plotted on Figures 5-1 and 5-2. With only  $\pm 10$  degree variation in pitch combined with  $\pm 30$  degree variation in roll we note that the heading display varies 14 degrees. Obviously,

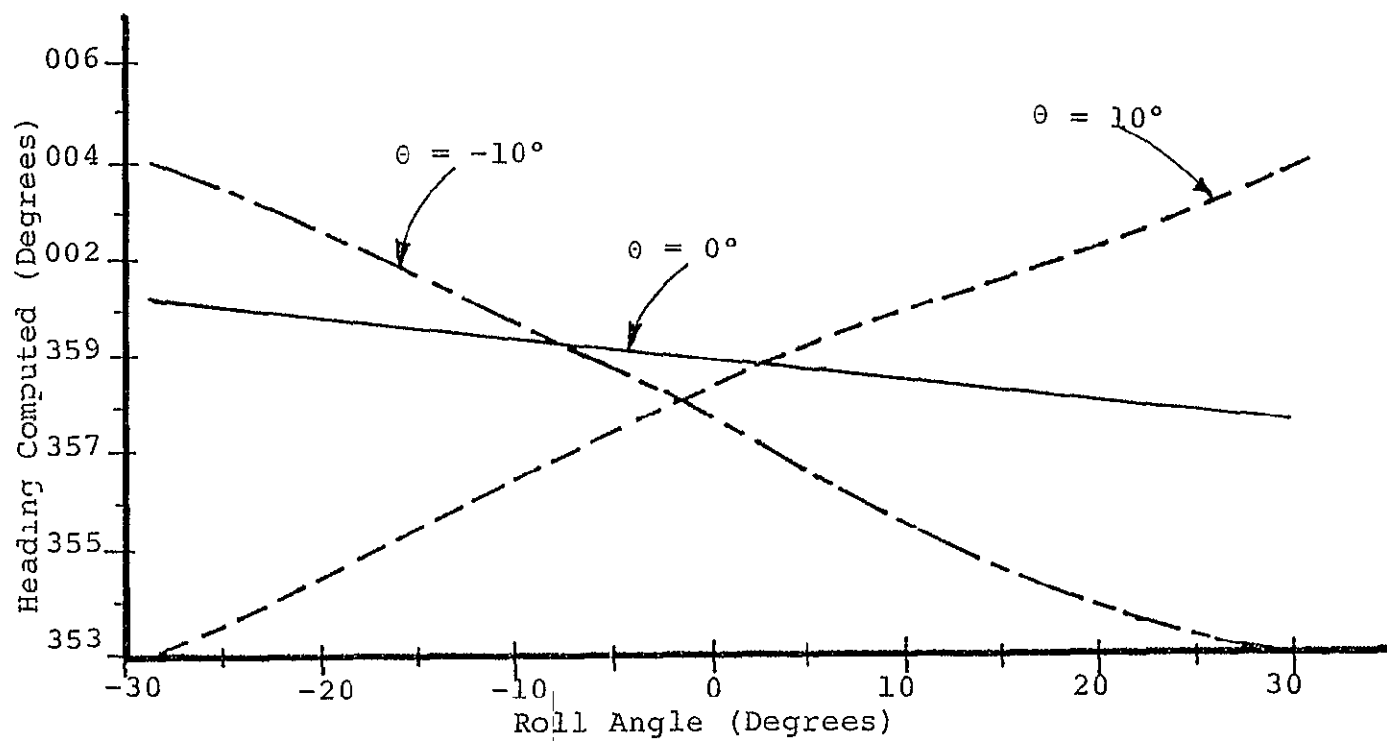


Fig. 5-1 HEADING COMPUTED AT A FIXED YAW ANGLE WITH VARYING  $\theta$  AND  $\phi$  (NO OFFSET CORRECTION)

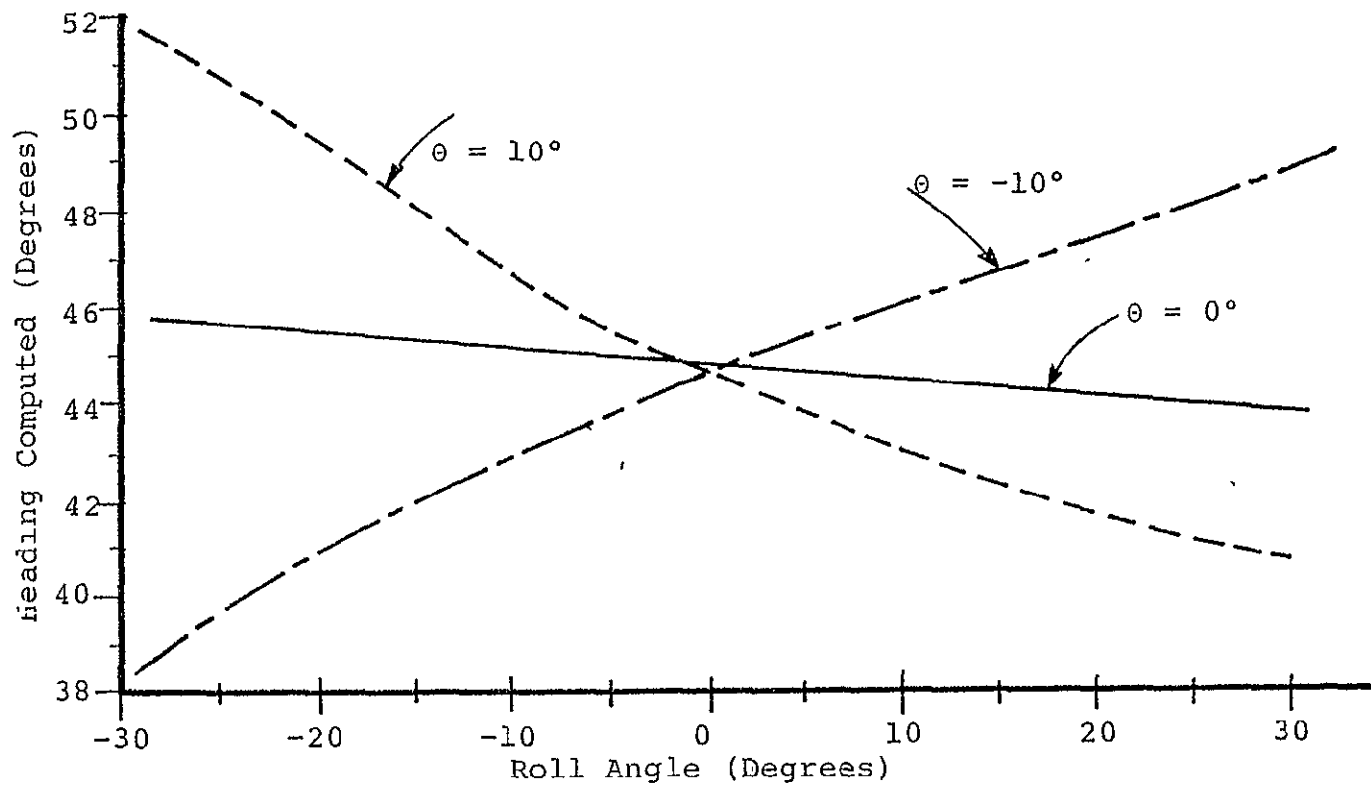


Fig. 5-2 HEADING COMPUTED AT A FIXED YAW ANGLE WITH VARYING  $\theta$  AND  $\phi$  (NO OFFSET CORRECTION)

instrument operation indicated excessive error requiring more elaborate sensors or correction of a sensor inadequacy.

#### 5-4 HEADING MEASUREMENTS TO INVESTIGATE ORTHOGONALITY ERROR

System performance was evaluated by initially aligning the sensors with zero pitch and roll angle. Sensor Z was positioned vertically with positive direction downwards. By observing the Z axis output<sup>2</sup> as the test fixture was rotated about the vertical axis, adjustments were made in pitch and roll angle to minimize coning of the Z axis. Angular measurements on the respective roll and pitch axis protractors were then made to establish the initial reference attitude angles.

Heading measurement accuracy was evaluated by rotating the test fixture in the horizontal plane until the display flickered between (XX9) and (XX9+1). The rotation was then continued a very small amount until a steady display (multiple of 10 degrees) was observed<sup>3</sup>. Measurements ranging from 0 to 350 degrees were made by recording angular position required to produce specific heading data displays. Sets of data were recorded at various combinations of pitch and roll then tabulated in Tables 5-3 through 5-8. Relative error was computed by determining angular position expected at each display value and then computing the difference in angular positions. Errors at the roll extremes of  $\pm 44$  degrees are plotted for pitch angles of plus and minus 20 degrees on Fig. 5-3 through 5-6 inclusive.

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<sup>2</sup>A special subroutine was used to display Z axis data directly in BCD format on the seven bar output display.

<sup>3</sup>This measurement technique ensured that all heading measurements were made identically. In addition, error due to system imprecision was reduced.

Data in Tables 5-5, 5-6 and Fig. 5-3, 5-4 were recorded with no sensor orthogonality error correction implemented. Data in Tables 5-7, 5-8 and Fig. 5-5, 5-6 was recorded with the sensor orthogonality correction implemented. Comparison of these data indicate that considerable improvement in accuracy is achieved by correcting sensor orthogonality error.

## 5-5 CONCLUSIONS

Laboratory evaluation of the heading measurement instrument has shown that the algorithms developed in previous chapters are viable. Operation of the device in a laboratory environment has enabled empirical evaluation of the system under adverse combinations of noise, field gradient and sensor plus instrument error sources.

Test apparatus described in section 5-2 served to enable controlled simulation of roll, pitch and yaw rotations. The apparatus was nonmagnetic in nature and contributed insignificant error due to field perturbation. Mounting of protractors and pointers on the test apparatus made angular measurements possible to a precision of at least  $\pm 0.5$  degrees.

Effects of sensor offsets were evaluated in section 5-3 by recording system heading computations when only roll and pitch varied. Since the variations in Figures 5-1 and 5-2 prior to offset correction exceed the maximum excursions of Figures 5-3 and 5-4 by at least a factor of two (angular excursions in first set also less than in the record) and we note that offset errors were corrected prior to recording data in the second set of data, we conclude that offset in magnetometers can be a

PITCH ANGLE 0 DEGREES

ROLL ANGLE 0 DEGREES

Heading Displayed (Degrees)	Angular Position (Degrees)	Relative Error (Degrees)
10	275.3	0.3
30	295.2	0.2
50	315.4	0.4
70	335.5	0.5
90	355.5	0.5
130	34.8	-0.2
150	54.3	-0.7
170	74.0	-1.0
190	94.0	-1.0
210	114.4	-0.6
230	135.0	0.0
250	155.5	0.5
270	174.7	-0.3
290	195.5	0.5
310	215.2	0.2
330	235.2	0.2
350	254.7	-0.3

Table 5-3    REFERENCE DATA MEASUREMENTS OF HEADING  
TAKEN WITH NO ORTHOGONALITY CORRECTION



PITCH ANGLE 0 DEGREES

ROLL ANGLE 0 DEGREES

Heading Displayed (Degrees)	Angular Position (Degrees)	Relative Error (Degrees)
10	276.6	-0.4
30	296.6	-0.4
40	306.7	-0.3
50	316.7	-0.3
60	327.2	+0.2
70	337.0	0.0
90	355.9	-1.1
130	37.0	0.0
160	67.2	0.2
190	96.8	-0.2
220	126.9	-0.1
250	157.0	0.0
280	186.9	-0.1
310	216.6	-0.4
340	246.9	-0.1
350	256.4	-0.6

Table 5-4    HEADING MEASUREMENTS WITH OFFSET AND  
ORTHOGONALITY CORRECTIONS MADE

Heading Displayed (Degrees)	Roll = 44°		Roll = 20°		Roll = -20°		Roll = -44°	
	Angular Position	Error	Angular Position	Error	Angular Position	Error	Angular Position	Error
10	276.3	1.3	276.5	1.5	275.0	0.0	274.0	-1.0
30	296.3	1.3					293.6	-1.4
40			306.1	1.1	304.0	-1.0		
50							313.8	-1.2
60	325.8	0.8						
70			335.6	1.6	334.3	-0.7	333.8	-1.2
90	355.0	0.0	354.0	-1.0	353.1	-1.9	353.0	-2.0
130	34.3	0.7	34.3	-0.7	34.6	-0.4	34.4	-0.6
160	64.5	-0.5	64.0	-1.0	65.0	0.0	65.0	0.0
190	94.3	-0.7	94.6	-0.4	95.0	0.0	95.5	0.5
220	125.2	0.2	125.3	0.3	126.0	1.0	126.5	1.5
250	156.6	1.6	156.2	1.2	156.7	1.7	157.0	2.0
280	186.6	1.6	186.8	1.8	186.5	1.5		
310	216.8	1.8	216.8	1.8	215.9	0.9	215.9	0.9
340	246.0	1.0	246.8	1.8	245.0	0.0		
350							224.9	-0.1

TABLE 5-5 HEADING MEASUREMENTS AT PITCH = 20° WITH NO  
ORTHOGONALITY CORRECTION

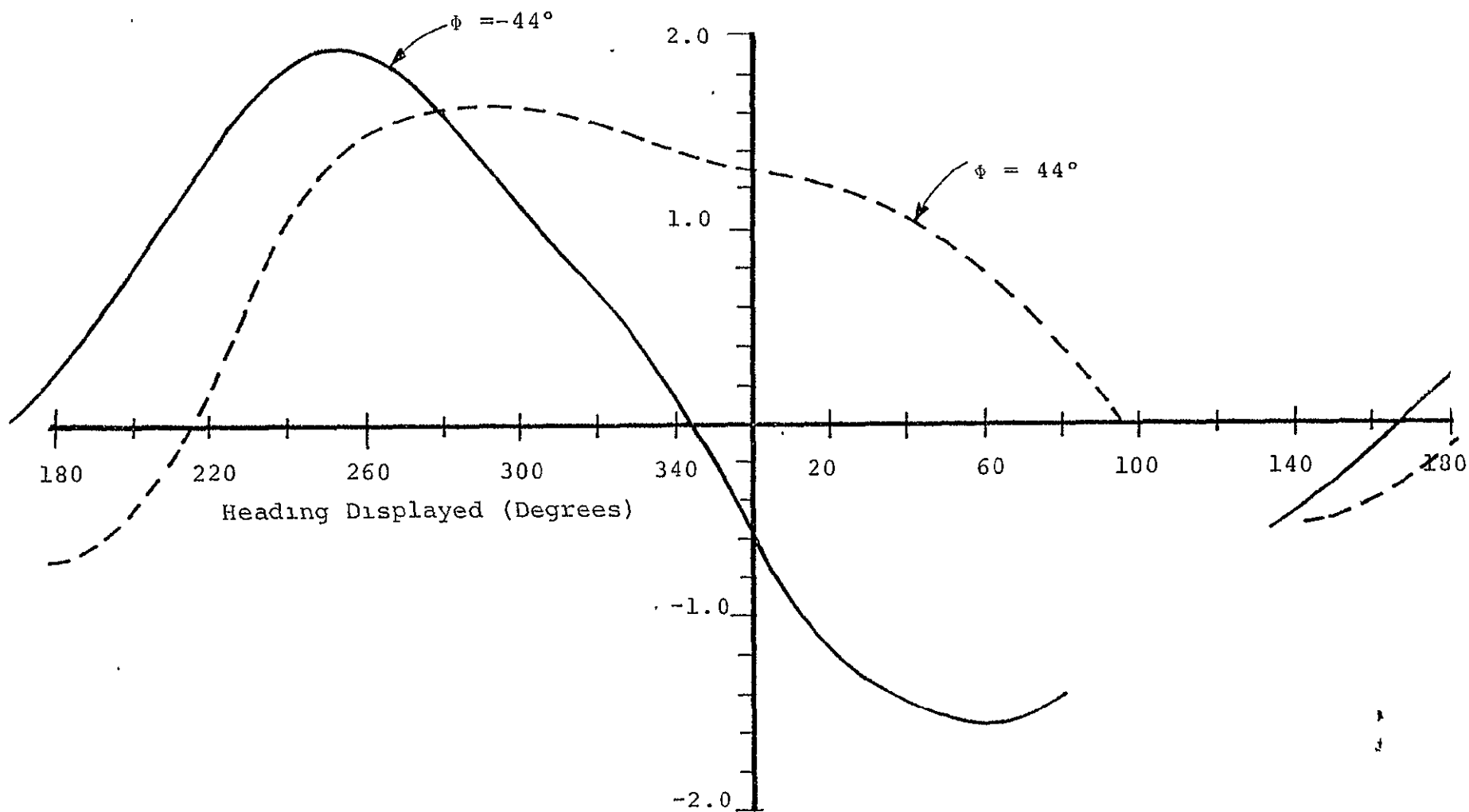


Fig. 5-3 HEADING MEASUREMENT ERROR AT PITCH =  $20^\circ$   
(NO ORTHOGONALITY CORRECTION)

Heading Displayed (Degrees)	Roll = 44°		Roll = 20°		Roll = -20°		Roll = -44°	
	Angular Position	Error	Angular Position	Error	Angular Position	Error	Angular Position	Error
10	276.1	1.1	275.1	0.1	274.2	-0.8	273.7	-1.3
30	296.2	1.2	295.9	0.9			293.6	-1.4
50	316.4	1.4	316.1	1.1			314.1	-0.9
70	336.5	1.5	336.4	1.4	335.5	0.5	334.6	-0.4
90	355.5	0.5	355.3	0.3	355.0	0.0	354.2	-0.8
130	36.0	1.0	36.4	1.4	135.7	0.7	135.7	0.7
150	55.3	0.3	55.9	0.9			55.8	0.8
170	74.5	-0.5	75.6	0.6			75.5	0.5
190	94.9	-0.1	95.3	0.3	95.5	0.5	95.5	0.5
210	115.0	0.0	115.4	0.4			115.6	0.6
220					125.5	0.5		
230	135.0	0.0	135.6	0.6			136.0	1.0
250	155.6	0.6	155.4	0.4	155.5	0.5	155.7	0.7
270	174.8	-0.2	174.8	-0.2			175.0	0.0
290	195.6	0.6	195.4	0.4			195.3	0.3
310	215.5	0.5	215.3	0.3	214.6	-0.4	214.5	-0.5
330	235.7	0.7	235.1	0.1			234.2	-0.8
350	255.6	0.6	255.2	0.2			253.8	-1.2

Table 5-6 HEADING MEASUREMENTS AT PITCH = -20°  
WITH NO ORTHOGONALITY CORRECTION

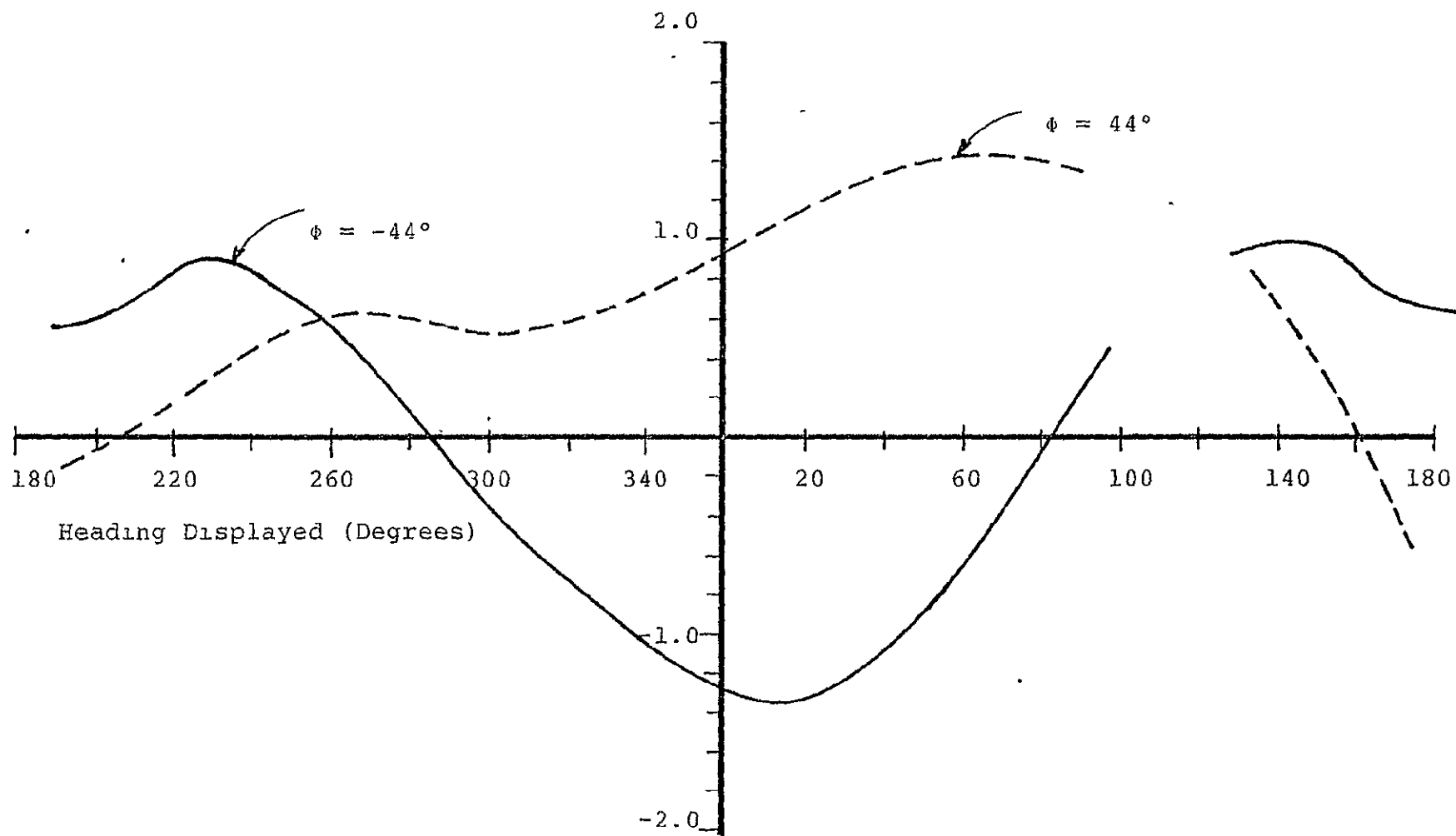


Fig. 5-4 HEADING MEASUREMENT ERROR AT PITCH =  $-20^\circ$   
(NO ORTHOGONALITY CORRECTION)

Heading Displayed (Degrees)	Roll = 0°		Roll = 44°		Roll = -44°	
	Angular Position	Error	Angular Position	Error	Angular Position	Error
20	287.3	0.3	287.5	0.3	286.3	-0.7
40	307.2	0.2	307.5	0.3	306.0	-1.0
60	327.0	0.0	327.9	0.7	325.9	-1.1
80	346.3	-0.7	347.5	0.3	345.6	-1.4
90	355.9	-1.1	357.0	0.2	356.2	-1.8
140	46.9	-0.1	47.0	-0.2	46.5	-0.5
160	67.0	0.0	67.3	0.1	67.1	0.1
180	86.6	-0.4	86.5	-0.7	87.0	0.0
200	107.0	0.0	107.3	0.1	108.1	1.1
220	127.6	0.6	127.3	0.1	128.3	1.2
240	147.2	0.2	147.4	0.2	148.5	1.5
260	167.0	0.0	167.1	-0.1	168.5	1.5
280	187.3	0.3	187.1	-0.1	188.5	1.5
300	207.2	0.2	207.1	-0.1	208.0	1.0
320	227.3	0.3	227.0	-0.2	227.5	0.5
340	247.4	0.4	247.3	+0.1	247.4	0.4
0	266.4	-0.6	267.0	-0.2	266.0	-1.0

Table 5-7    HEADING MEASUREMENTS AT PITCH = 20°  
 WITH OFFSET AND ORTHOGONALITY CORRECTION  
 MADE

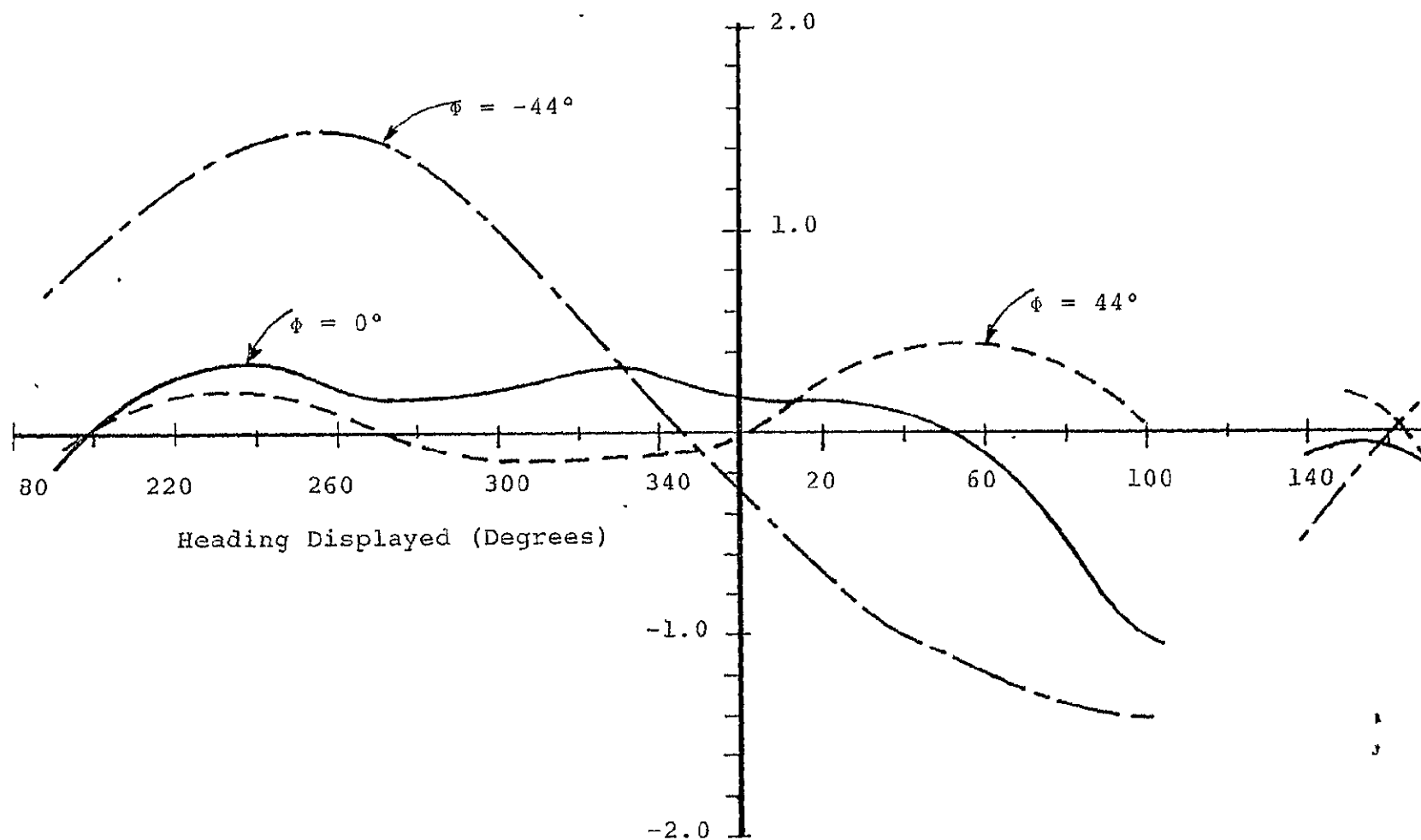


Fig. 5-5 HEADING MEASUREMENT ERROR AT PITCH -  $20^\circ$   
(OFFSET AND ORTHOGONALITY ERROR CORRECTED)

Heading Displayed (Degrees)	Roll = 44°		Roll = -44°	
	Angular Position	Error	Angular Position	Error
20	287.0	-0.4	286.5	-0.3
40	307.0	-0.4	306.7	-0.1
60	327.0	-0.4	326.8	0.0
80	347.0	-0.4	346.8	0.0
90	356.7	-0.7	355.6	-0.2
140	47.8	0.4	48.0	1.2
160	67.7	0.3	68.0	1.2
180	87.0	-0.4	87.8	1.0
200	107.6	0.2	108.0	1.2
220	127.7	0.3	128.0	1.2
240	147.8	0.4	147.4	0.6
260	167.6	0.2	167.2	0.4
280	187.6	0.2	187.0	0.2
300	207.1	-0.3	206.5	-0.3
320	227.1	-0.3	226.6	-0.2
340	247.1	-0.3	246.9	0.1
0	266.3	-0.9	266.0	-0.8

TABLE 5-8 HEADING MEASUREMENTS AT PITCH = -20° WITH  
OFFSET AND ORTHOGONALITY ERROR CORRECTED



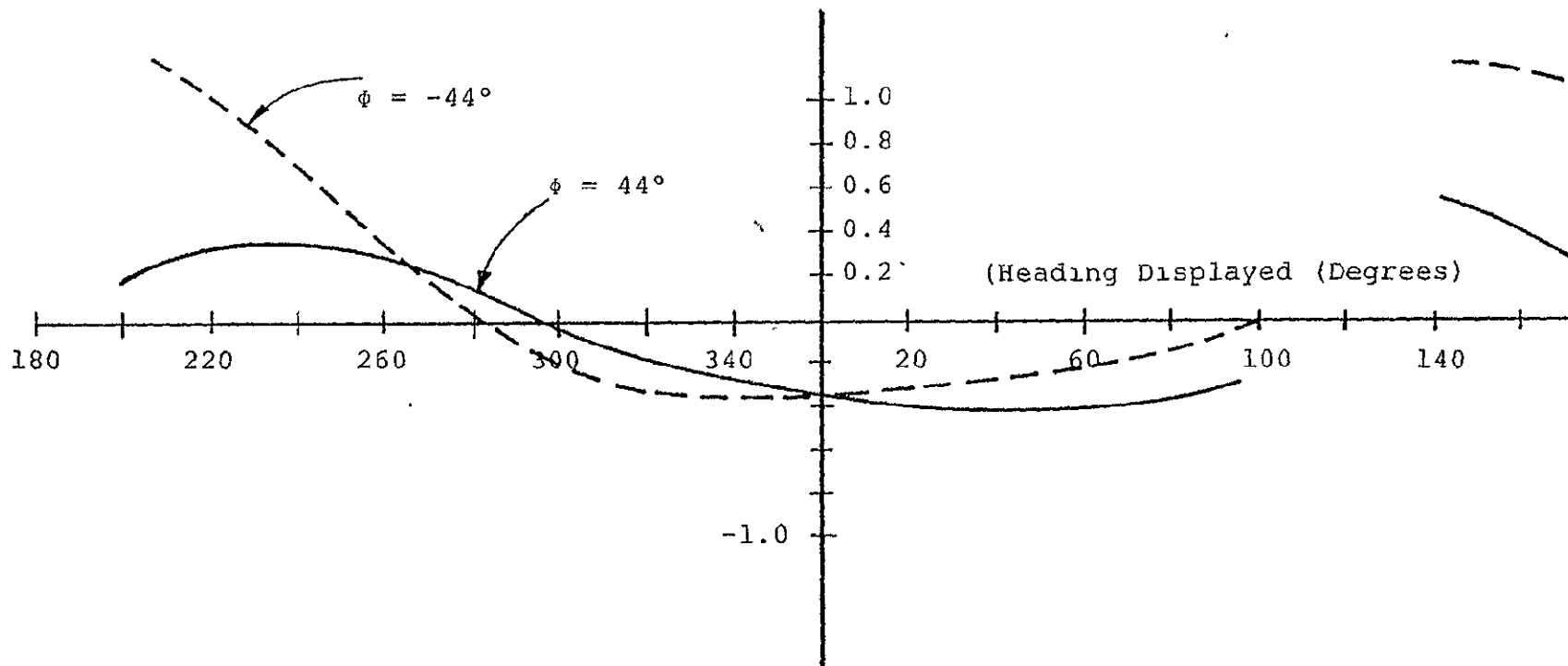


Fig. 5-6 HEADING MEASUREMENT ERROR AT PITCH =  $-20^\circ$   
(OFFSET AND ORTHOGONALITY CORRECTED)

major error source<sup>4</sup>. Additionally, we note that the correction of offset error in sensors has been successful. Experimental results have verified that not only can offset errors be determined (Chapter IV), but a suitable algorithm can be implemented in the computer to improve system operation. It is postulated that offset error correction can be extended to include correction of varying offset values (functions of temperature and supply voltage) by monitoring error causing variables (example temperature) and computing correction constants prior to offset correction as above.

Errors induced by sensor nonorthogonality were predicted in Chapter IV section 4-2 and verified by plotting expected error along with measured error in Fig. 4-14. The curves of Fig. 4-14 were plotted for heading rotations with no pitch or roll angle. To evaluate system performance and the effect of orthogonality error with combined angular rotations, measurements of heading error were plotted in Fig. 5-3 through 5-6 inclusive.

Comparison of these data indicate that maximum excursions of error as a function of heading are significantly less when orthogonality corrections are made. It is also postulated that data could be improved further by similarly correcting orthogonality error in the Z axis sensor<sup>5</sup>.

In summary, the experimental evaluation has provided insight into the operation of an attitude independent remote magnetic indicator and heading computer in the "real world"

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<sup>4</sup>This corroborates the observations predicted during error analysis in Chapter IV.

<sup>5</sup>We note that the error excursions are functions of pitch and roll and that Z axis data is used in the rotation algorithm.

environment complete with all contributing error sources. The error analysis evolved during development of the system has proven adequate in that an operational system was developed. Major error sources were measurable as predicted and the means of reducing their effects were successfully implemented. Correction of sensor offset and orthogonality error required an empirical evaluation of the respective sensor. These evaluations were performed, the errors characterized, correction coefficients determined, and correction algorithms implemented.

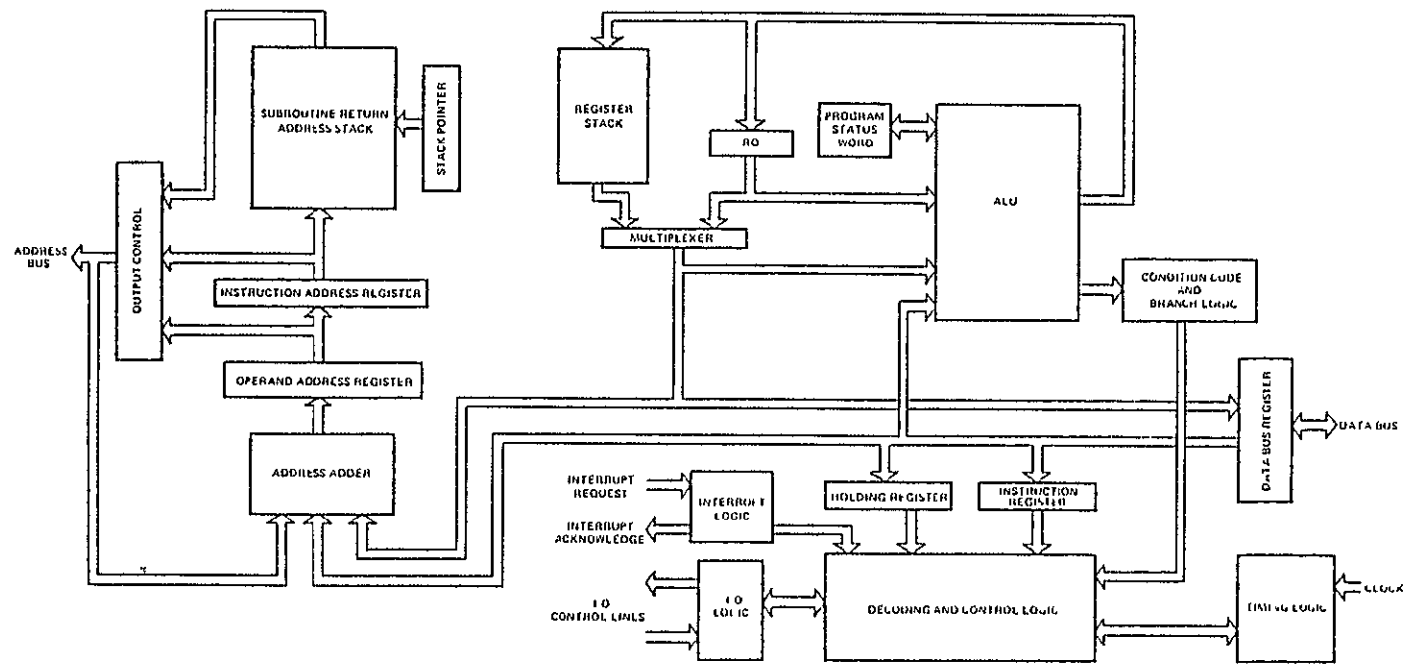
Successful implementation of these corrections was evidenced by significant reductions in system error. The correction methods presented can be extended in future with the net result that less demand is required of physical sensors if the sensor parameters can be established empirically prior to completion of instrument design. Utilization of a microprocessor in the instrument has added the computational flexibility required to facilitate accommodation of sensors with varying error magnitudes.

## APPENDIX A

This appendix lists the instruction set of the Signetics 2650 microprocessor chip used to implement the heading instrument.

LOAD/STORE INSTRUCTIONS			Length (bytes)			
LODZ	r	Load Register Zero	1		BIR \,r (*j)a	Branch on Incrementing Register Absolute 3
LODI r	v	Load Immediate	2		BDRR,r (*j)a	Branch on Decrementing Register Relative 2
LODR,r	(*j)a	Load Relative	2		BDRR,r (*j)a	Branch on Decrementing Register Absolute 3
LODA,r	(*j)a(X)	Load Absolute	3		BXA (*j)a(X)	Branch Indexed Absolute Unconditional 3
STRZ	r	Store Register Zero	1		ZBRr (*j)a	Zero Branch Relative, Unconditional 2
STRR,r	(*j)a	Store Relative	2			
STR A,r	(*j)a(X)	Store Absolute	3			
ARITHMETIC INSTRUCTIONS						
ADDZ	r	Add to Register Zero	1			
ADDI r	v	Add Immediate	2			
ADDR,r	(*j)a	Add Relative	2			
ADD \,r	(*j)a(X)	Add Absolute	3			
SUBZ	r	Subtract from Register Zero	1			
SUBI r	v	Subtract Immediate	2			
SUBR,r	(*j)a	Subtract Relative	2			
SUBA,r	(*j)a(X)	Subtract Absolute	3			
LOGICAL INSTRUCTIONS						
ANDZ	r	And to Register Zero	1			
ANDI r	v	And Immediate	2			
ANDR,r	(*j)a	And Relative	2			
ANDA,r	(*j)a(X)	And Absolute	3			
IORZ	r	Inclusive or to Register Zero	1			
IORI,r	v	Inclusive or Immediate	2			
IORR,r	(*j)a	Inclusive or Relative	2			
IOR \,r	(*j)a(X)	Inclusive or Absolute	3			
EORZ	r	Exclusive or to Register Zero	1			
EORI r	v	Exclusive or Immediate	2			
EORR,r	(*j)a	Exclusive or Relative	2			
EOR \,r	(*j)a(X)	Exclusive or Absolute	3			
COMPARISON INSTRUCTIONS						
CONZ	r	Compare to Register Zero	1			
CONI r	v	Compare Immediate	2			
COMR,r	(*j)a	Compare Relative	2			
COM \,r	(*j)a(X)	Compare Absolute	3			
ROTATE INSTRUCTIONS			Length (bytes)			
RRR,r		Rotate Register Right	1			
RLR,r		Rotate Register Left	1			
BRANCH INSTRUCTIONS						
BCIR,v	(*j)a	Branch on Condition True Relative	2			
BCIR v	(*j)a	Branch on Condition False Relative	2			
BCIA,v	(*j)a	Branch on Condition True Absolute	3			
BCF \,v	(*j)a	Branch on Condition False Absolute	3			
BRNR,r	(*j)a	Branch on Register Non-Zero Relative	2			
BRN \,r	(*j)a	Branch on Register Non-Zero Absolute	3			
BIRR,r	(*j)a	Branch on Incrementing Register Relative	2			
SUBROUTINE BRANCH/RETURN INSTRUCTIONS						
BSIR,v	(*j)a	Branch to Subroutine on Condition True, Relative	2			
BSIR,v	(*j)a	Branch to Subroutine on Condition False, Relative	2			
BSIA,v	(*j)a	Branch to Subroutine on Condition True Absolute	3			
BSFA,v	(*j)a	Branch to Subroutine on Condition False Absolute	3			
BSNR,r	(*j)a	Branch to Subroutine on Non Zero Register, Relative	2			
BSNA,r	(*j)a	Branch to Subroutine on Non Zero Register, Absolute	3			
BSXA (*j)a(X)		Branch to Subroutine Indexed Unconditional	3			
RETC,v		Return From Subroutine Conditional	1			
RETE,v		Return From Subroutine and Enable Interrupt, Conditional	1			
ZBSR (*j)a		Zero Branch to Subroutine Relative, Unconditional	2			
PROGRAM STATUS INSTRUCTIONS						
LPSU		Load Program Status, Upper	1			
LPSL		Load Program Status, Lower	1			
SPSU		Store Program Status, Upper	1			
SPSL		Store Program Status, Lower	1			
CPSU v		Clear Program Status, Upper Selective	2			
CPSL v		Clear Program Status, Lower Selective	2			
PPSU v		Preset Program Status, Upper Selective	2			
PPSL v		Preset Program Status, Lower Selective	2			
TPSU v		Test Program Status Upper Selective	2			
TPSL v		Test Program Status Lower Selective	2			
INPUT/OUTPUT INSTRUCTIONS						
WRD,r		Write Data	1			
RED,r		Read Data	1			
WRC,r		Write Control	1			
REC,r		Read Control	1			
WRTE,r v		Write Extended	2			
REDE,r v		Read Extended	2			
MISCELLANEOUS INSTRUCTIONS						
HAIT		Halt Enter Wait State	1			
DAR r		Decimal Adjust Register	1			
TMI r v		Test Under Mask Immediate	2			
NOF		No Operation	1			

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BLOCK DIAGRAM

## APPENDIX B

This appendix contains a listing of the assembly language program used to implement the remote magnetic indicator heading algorithm. The program was assembled on the A2650 cross assembler program operational on the HP 2100 computer at the University of Santa Clara.

LINE	ADDR	LEVEL	B1	B2	B3	B4	ERROR	SOURCE
3								*****
4	0000							R0 EQU 0
5	0001							R1 EQU 1
6	0002							R2 EQU 2
7	0003							R3 EQU 3
8	0000							UN EQU 0
9	0000							Z EQU 0
10	0001							P EQU 1
11	0002							N EQU 2
12	0000							FQ EQU 0
13	0001							GT EQU 1
14	0001							C EQU 1
15	0002							LT EQU 2
16	0003							UN EQU 3
17	0002							COM EQU H'02'
18	0010							HS EQU H'10'
19	0008							WC EQU H'08'
20	000F							LOWR EQU H'0F'
21	00F0							UPPR EQU H'F0'
22	0040							FLAG EQU H'40'
23	0020							II EQU H'20'
24	00FF							NSAM EQU H'FF'
25	0001							MSB EQU 1
26	0002							LSB EQU 2
27	0004							THTA EQU 4
28	0008							PHI EQU 8
29	0010							SBIT EQU 16
30								*****
31								DATA STORAGE IN RAM
32								ORG H'450'
33	0450							DATA RES 10
34	045A							DATM RES 10
35	0464							LUPS RES 1
36	0465							TEMP RES 2
37	0467							SIND RES 2
38	0469							SINR RES 2
39	046B							HM2 RES 2
40	046D							SIGN RES 1
41	046E							HX2 RES 2
42	0470							HY2 RES 2
43	0472							HZ2 RES 2
44	0474							QUAD RES 1
45	0475							COSR RES 2
46	0477							WSLT RES 2
47	0479							OPR1 RES 2
48	047B							OPR2 RES 2
49	047D							STEM RES 1
50	047E							SHES RES 1
51	047F							DATX RES 2
52	0481							DATY RES 2
53	0483							DATZ RES 2
54	0485							COST RES 1

OUTPUT  
OUTPUT

RAW DATA  
CORRECTED DATA  
  
HM SQUARED DATA  
HX SQUARED DATA

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LINE	ADDR	FILE	01	02	03	04	ERROR	SOURCE
55		0446						SIGX RES 1
56		0447						SIGY RES 1
57		0448						SIGZ RES 1
58		0449						ABUF RES 1
59		044A						BBUF RES 1
60		044B						CBUF RES 1
61		044C						DBUF RES 1
62		044D						THUF RES 1
63		044E						0SET RES 6
64		044F						0OST RES 6
65								*****
66								CONSTANTS IN WORD *****
67								ORG H:5000
68	0500	0500	00	01	03	04		SIN DATA H:100,01,04,06,07,09,0A,0C,0E,0F
			06	07	09	0A		
			0C	0E	0F			
69	0504		11	12	14	15		DATA H:11,12,14,15,17,19,1A,1C,1D,1F,20
			17	19	1A	1C		
			1D	1F	20			
70	0516		22	24	25	27		DATA H:22,24,25,27,28,2A,2B,2D,2E,30,31
			28	2A	2B	2D		
			2E	30	31			
71	0521		33	35	36	38		DATA H:33,35,36,38,39,3B,3C,3F,41,42
			39	3B	3C	3F		
			3F	41	42			
72	052C		44	45	47	48		DATA H:44,45,47,48,4A,4B,4D,4F,50,51,53
			4A	4B	4D	4F		
			50	51	53			
73	0537		55	56	57	59		DATA H:55,56,57,59,5A,5C,5D, 01 DEF
			5A	5C	5D			
74	053F		5F	60	61	63		DATA H:5F,60,61,63,64,66,67,69,6A,6C,6D
			64	66	67	69		
			6A	6C	6D			
75	0549		6E	70	71	73		DATA H:6E,70,71,73,74,75,77,78,7A,7B,7C
			74	75	77	78		
			7A	7B	7C			
76	0554		7E	7F	80	82		DATA H:7E,7F,80,82,83,84,86,87,8A,8B,8C
			83	84	86	87		
			8A	8B	8C			
77	0560		8E	8F	90	92		DATA H:8E,8F,90,92,93,94,95,97,99,9A,9B
			93	94	95	97		
			9A	9B				
78	0568		9C	9D	9E	9F		DATA H:9C,9D,9E,9F,A1,A2,A3,A4,A6,A7,A8
			A1	A2	A3	A4		
			A6	A7	A8			
79	0576		AA	AB	AD	AE		DATA H:AA,AB,AD,AE,AF,81,83,85
			AB	AD	AE	AF		
			81	83	85			
80	0581		88	89	8A	8B		DATA H:88,89,8A,8B,8C,8D,8F,90,91,92
			8A	8B	8C	8D		
			8F	90	91	92		
81	058D		8C	8D	8E	8F		DATA H:8C,8D,8E,8F,91,92,93,94,95,96,97
			8D	8E	8F	91		
			92	93	94	95		

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LINE	ADDR	LABL	B1 B2 B3 B4	ERROR	SOURCE	
82	0598		CE CF 00 D1 D2 D3 D3 D4 D5 D6 D7		DATA	H:CE,CF,D0,D1,D2,D3,D3,D4,D5,D6,D7
83	05A3		D8 D9 D9 DA DB DC DD DD DE DF E0		DATA	H:DB,D9,D9,DA,DB,DC,DD,DD,DE,DF,E0
84	05AE		F1 E1 C2 E3 E3 E4 E5 E6 E6 E7 E8		DATA	H:E1,E1,E2,E3,E3,E4,E5,E6,E6,E7,E8
85	05B9	05B9	FF FF FF FF FF FF FF FF FF FF	CUS	DATA	H:FF,FF,FF,FF,FF,FF,FF,FF,FF,FF
86	05C3		FF FF FF FF FF FF FE FF FE FE		DATA	H:FF,FF,FF,FF,FF,FE,FE,FE,FE,FE
87	05CD		FE FD FD FD FD FC FC FC FC FC		DATA	H:FE,FD,FD,FD,FD,FC,FC,FC,FC,FC
88	05D7		FB FB FB FA FA FA F9 F9 F9 FA		DATA	H:FB,FB,FB,FA,FA,FA,F9,F9,F9,F8
89	05E1		FB F7 F7 F7 FA F6 F5 F5 F4 F4		DATA	H:FB,F7,F7,F7,F6,F6,F5,F5,F4,F4
90	05EB		F4 F3 F3 F2 F2 F1 F1 F0 EF EF		DATA	H:F4,F3,F3,F2,F2,F1,F1,F0,EF,EF
91	05F5		EE EF FD ED EC EB EB EA EA E9 E8 E8		DATA	H:EE,FE,ED,ED,EC,EB,EB,EA,EA,E9,EB,EB
92	0601		E7 E6 E6 E5 E4 E3 E3 E2 E1 E1		DATA	H:E7,E6,E6,E5,E4,E3,E3,E2,E1,E1
93	060B		E0 DF DE DD DD DC DB DA D9 D9		DATA	H:E0,DF,DF,DD,DD,DC,DB,DA,D9,D9
94	0615		DB D7 D6 D5 D4 D3 D3 D2 D1 D0		DATA	H:DB,D7,D6,D5,D4,D3,D3,D2,D1,D0
95	061F		CF CF CD CC CB CA C9 CA C7 C4		DATA	H:CF,CE,CD,CC,CB,CA,C9,CA,C7,C6
96	0629		C5 C4 C3 C2 C1 C0 BF BE BD BC		DATA	H:C5,C4,C3,C2,C1,C0,BF,BE,BD,BC
97	0633		BB BA B9 B8 B7 B6 B5 B3 B2 B1		DATA	H:BB,BA,B9,B8,B7,B6,B5,B3,B2,B1
98	063D		B0 AF AE AN AB AA A9 AA A7 A4 A4		DATA	H:B0,AF,AE,AD,AB,AA,A9,AA,A7,A6,A4
99	0648		A3 A2 A1 9F 9E 9D 9C 9B 99 94 97 95		DATA	H:A3,A2,A1,9F,9E,9D,9C,9B,99,94,97,95

LINE	ADDR	LAHL	B1	B2	B3	B4	FROM	SOURCE
100	0654		94	93	92	90		DATA
			8F	8E	8C	8A		H:94,93,92,90,8F,8E,8C,8A,88,
			8A	8A				
101	065E		87	86	84	83		DATA
			82	80	7F	7E		H:87,86,84,83,82,80,7F,7E,7C,7A,
			7C	7B				
102	0668		7A	78	77	75		DATA
			74	73	71	70		H:7A,78,77,75,74,73,71,70,6E,6D,6C,
			6E	6D	6C			
103	0673		6A	69	67	6A		DATA
			64	63	61	60		H:6A,69,67,66,64,63,61,60,5F,5D,5C,
			5F	5D	5C			
104	067E		5A	59	57	5A		DATA
			54	53	51	50		H:5A,59,57,56,54,53,51,50,4F,4D,
			4E	4D				
105	0688		4B	4A	48	47		DATA
			45	44	42	41		H:4B,4A,48,47,45,44,42,41,3F,3E,3C,
			3F	3E	3C			
106	0693		3B	39	38	3A		DATA
			35	33	31	30		H:3B,39,38,36,35,33,31,30,2E,2D,
			2E	2D				
107	069D		2B	2A	28	27		DATA
			25	24	22	20		H:2B,2A,28,27,25,24,22,20,1F,1D,
			1F	1D				
108	06A7		1C	1A	19	17		DATA
			15	14	12	11		H:1C,1A,19,17,15,14,12,11,0F,0E,
			0F	0E				
109	06B1		0C	0A	09	07		DATA
			06	04	03	01		H:0C,0A,09,07,06,04,03,01,00,
			00					
110	06BA	06BA	7F	7B	77	72		COSM DATA
			6E	69	65	61		H:7F,7B,77,72,6E,69,65,61,5C,58,54,
			5C	58	54			
111	06C5		50	4A	47	43		DATA
			3F	3C	38	34		H:50,4A,47,43,3F,3C,38,34,2D,2A,
			31	2D	2A			
112	06D0		27	23	20	1D		DATA
			1B	18	15	13		H:27,23,20,1D,1B,18,15,13,0E,0C,
			11	0F	0C			
113	06DB		0B	09	07	0A		DATA
			04	03	02	01		H:0B,09,07,06,04,03,02,01,00,00,00,00,
			01	00	00	00		
			00					
114	06EB	06FB	FF	8B	12	9F		COSL DATA
			2F	C5	63	0A		H:FF,8B,12,9E,2F,C5,63,0A,B7,72,38,0C,
			47	72	38	0C		
115	06F4		F0	E3	F8	FF		DATA
			2B	6C	C3	32		H:F0,E3,F8,FF,2B,6C,C3,32,B9,59,15,EC,
			49	59	15	EC		
116	0700		F0	F2	F2	F2		DATA
			F2	F2	F2	F2		H:F0,F2,F2,F2,F2,F2,F2,F2,10,52,DB,
			F4	1	52	DB		

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LINE	ADDR	LAHL	B1	B2	B3	B4	ERROR	SOURCE
117	070C		43	F5	CD	CC		DATA H:43,F5,CD,CC,F1,3E,B3,4F,13,00
			F1	3E	B3	4F		
			13	00				
118	0716	0716	01	02	04	08		BCD DATA H:01,02,04,08,16,32,64,00
			16	32	64	00		
119	071E	071E	03					SINE DATA H:03
120								*****
121								ORG H:750
122								*****
123								* INITIALIZATION OF THE INSTRUMENT *
124								* PARAMETERS AFFECTED INCLUDE: *
125								* (1) PH. STATUS LOWER/UPPER *
126								* (2) CONTENTS OF REGS. C&D ON CPU BOARD *
127								*****
128								**** INITIALIZE ****
129	0750		76	21				PPSU II INHIBIT INTERRUPTS
130	0752		75	FF				CPSL H:FF CLEAR ALL
131	0754		77	02				PPSL COM LOGICAL COMPARISONS
132	0756		04	FF				LODI,R0 NSAM
133	0758		F0					WRTU,R0 NO SAMPLE/HLD OR MIX
134	0759		74	FF				CPSL H:FF CLEAR ALL
135								**
136								**** START MAIN PROGRAM ****
137	075B	075B	7F	07	B0			MAIN BSTA,UN SAMP SAMPLE ALL DATA CHANNELS
138	075E		7F	0A	E6			BSTA,UN ROTX CORRECT HX DATA
139	0761		7F	0A	71			BSTA,UN ROTY CORRECT HY DATA
140	0764		7F	0A	09			BSTA,UN HVEL COMPUTE HORIZ. VECTOR
141	0767		7F	07	70			BSTA,UN WICH CALC. HEADING & O/P
142	076A		3F	0B	57			BSTA,UN OUTA O/P ROLL & PITCH
143	076D		1F	07	5B			BSTA,UN MAIN LOOP FOREVER
144								*****
145								*
146								*
147	0770	0770	77	02				WICH PPSL LOM
148	0772		0C	04	70			LODA,R0 HY2
149	0775		EC	04	6E			COMA,R0 HX2
150	0778		10	0B	14			BCTA,GT COSY
151	077B		1E	0B	2B			BCTA,LT SINY
152	077E		0C	04	71			LODA,R0 HY2+1 LS BITS
153	0781		EC	04	6F			COMA,R0 HX2+1
154	0784		10	0B	14			BCTA,GT COSY
155	0787		1F	0B	2B			BCTA,UN SINY
156								*****

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LINE	ADDR	LA-L	H1	B2	B3	B4	ERROR	SOURCE
158								ORG H17H01
159								*****
160								* A SUBROUTINE TO SAMPLE DATA FROM ALL SENSORS *
161								* REGISTER "D" CONTROLS SAMPLE/HOLD & MUX *
162								* BITS 0,1,&2 SELECT HX, HY, HZ, THETA, PHI *
163								* DATA IS STORED IN DOUBLE PRECISION FORM *
164								* BEGINNING AT "DATA" WITH HX(MSB) FOLLOWED BY *
165								* HX(LSB), HY, HZ, PITCH, ROLL. *
166								* (1) CONVERTS A/D DATA TO SIGN MAGNITUDE *
167								* (2) CHANGES SIGN OF HY & HZ(ORIENTATION) *
168								* (3) CORRECTS FOR OFFSET ERROR *
169								* EXIT WITH CORRECTED DATA IN TABLE "DATA" *
170								*****
171								**** BEGIN SUBROUTINE ****
172	07H0	07H0	05	FF				SAMP LODI,R1 -1 DATA INDEX
173	07H2		06	05				LODI,R2 5 LOOP COUNTER
174	07H4		07	01				LODI,R3 1 1ST SAMPLE
175	07H6		75	09				CPSL C+WC
176	07H8	07H8	F3					NEXT WRTI,R3 SELECT & SAMPLE DATA
177	07H9		67	80				IORI,R3 H1801 READY TO HOLD
178	07HB		F3					WRTI,R3 HOLD DATA
179	07HC		04	80				LODI,R0 H1801
180	07HE		00					WRTI,R0 RESET A/D
181	07HF		20					EORZ R0
182	07L0		00					WRTI,R0 START A/D
183	07C1	07C1	70					TEST REDU,R0 READ LSB
184	07C2		F4	01				T4I,R0 H1011
185	07C4		1A	7H				BCTR,N TEST
186	07C6		24	FF				EORI,R0 H1FF1 BRANCH BACK IF FOR=1
187	07C8		44	FD				ANDI,R0 UPP4
188	07CA		00	24	51			STRA,R0 DATA+1,R1,+ STORE LS 1/3
189	07CD		30					HEUC,R0 READ MS 2/3
190	07CF		24	FF				EORI,R0 H1FF1
191	07D0		00	24	4F			STRA,R0 DATA-1,R1,+ STORE MSB
192	07D3		07	01				ADDI,R3 1 POINT TO NEXT DATA
193	07D5		47	0F				ANDI,R3 LOWH POINT TO CHAN, ONLY
194	07D7		FA	5F				WDRH,R2 NEXT TEST LOOP COUNTER
195								*****
196								* CONVERT TO SIGN MAGNITUDE
197	07D9		77	00				PPSL MC ENAHLE CARRY
198	07DB		06	FF				LODI,R2 -1 INDEX
199	07DD		07	07				LODI,R3 7 LOOP COUNTER
200	07DF	07DF	0E	24	50			LODA,R0 DATA,R2,+ GET MSB OF DATA
201	07E2		1A	17				BCTR,N H510 BR. IF SIGN=1
202	07E4		C1					SINZ R1
203	07E5		20					EORZ R0
204	07E6		77	01				PPSL C
205	07E8		AE	24	50			SUBA,R0 DATA,R2,+ SUB. LS BITS
206	07EB		CE	64	5A			STRA,R0 DATA,R2 STORE NEW DATA
207	07EE		20					EORZ R0
208	07EF		A1					SUBZ R1
209	07F0		64	80				IORI,R0 H1801 SET SIGN=1 (NFG)

LINE	ADDR	LABL	H1	B2	H3	B4	ERROR	SOURCE
210	07F2		CE	64	59			STRA,R0 DATM-1,R2 STORE MS BITS
211	07F5		18	08				BCTH,UN FINI
212	07F7	07F7	44	7F				RSTO ANDI,R0 H+7F SIGN BIT = 0
213	07F9		CE	64	5A			STRA,R0 DATM,R2 STORE NEW DATA
214	07FC		0E	24	50			LDDA,R0 DATA,R2,+ GET LS DATA
215	07FF		CE	64	5A			STRA,R0 DATM,R2 STORE AS NEW DATA
216	0802	0802	FB	58				FINI BDRH,R3 CONT
217								" CHANGE SIGN OF HY & HZ (SENSOR ORIENTATION)
218	0804		0C	04	5C			LDDA,R0 DATM+2 CHANGE SIGN OF HY & HZ
219	0807		0D	04	5E			LDDA,R1 DATM+4
220	080A		24	80				EORI,R0 H+80
221	080C		25	80				EORI,R1 H+80
222	080E		CC	04	5C			STRA,R0 DAT+2
223	0811		CD	04	5E			STRA,R1 DATM+4
224								" CORRECT FOR OFFSET ERROR IN HX, HY, & HZ CHANNELS
225	0814		07	00				LDDI,R3 0 DO HX 1ST
226	0816		3F	08	2A			BSTA,UN OFST
227	0819		07	02				LDDI,R3 2 HY
228	081B		3F	08	2A			BSTA,UN OFST
229	081E		07	04				LDDI,R3 4 HZ
230	0820		3F	08	2A			BSTA,UN OFST
231	0823		C0					NOP
232	0824		07	07				LDDI,R3 7
233								" CORRECT FOR ORTHOGONALITY ERROR IN HX AXIS
234	0826		3F	0F	28			BSTA,UN ORTH
235	0829		17					RETC,UN
236								*****

LINE ADDR LABEL B1 B2 B3 B4 FROM SOURCE

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238
239
240 0B2A 0B2A 0F 64 5A
241 0B2B 01
242 0B2E 44 7F
243 0B30 45 70
244 0B32 CC 04 77
245 0B35 CD 04 7F
246 0B38 0F 64 5A
247 0B3B CC 04 7B
248 0B3F 0F 64 5E
249 0B41 CC 04 65
250 0B44 0F 64 5F
251 0B47 CC 04 66
252 0B4A 0F 64 54
253 0B4D CC 04 7D
254 0B50 75 02
255 0B52 77 70
256 0B54 7F 04 5B
257 0B57 75 70
258 0B59 0C 04 77
259 0B5C 0F 64 5A
260 0B5F 0C 04 7B
261 0B62 0F 64 5B
262 0B65 17
263
264
265
266
267
268
269
270
271
272 0B66 0B66 77 0A
273 0B68 20
274 0B69 CC 04 77
275 0B6C CC 04 7B
276 0B6F CC 04 79
277 0B72 07 0B
278 0B74 0B74 0E 04 7A
279 0B77 0B77 52
280
281 0B7A 70
282 0B79 00
283 0B7A 7B 00
284 0B7C 0B7C 05 02
285 0B7F 0B7F 00 64 76
286 0B81 00 64 7A
287 0B84 00 64 76
288 0B87 79 75
289

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*****
OFST  LODA,R0  DATM,R3  MS BYTE OF DATA
      STRZ     R1
      ANDI,R0  M1/7F    STRIP SIGN
      ANDI,R1  M1B0     SAVE SIGN
      STRA,R0  RSLT
      STRA,R1  SRES
      LODA,R0  DATM+1,R3  LS BYTE OF DATA
      STRA,R0  RSLT+1
      LODA,R0  OSET,R3  MS BYTE OF OFFSET
      STRA,R0  TEMP
      LODA,R0  OSET+1,R3
      STRA,R0  TEMP+1
      LODA,R0  SOST,R3
      STRA,R0  STE 1
      CPSL     CNM
      PPSL     KS  BANK=1
      BSTA,UN  SADI
      CPSL     KS  BANK=0
      LODA,R0  RSLT
      STRA,R0  DATM,R3
      LODA,R0  RSLT+1
      STRA,R0  DATM+1,R3
      RETC,UN
*****
* BINARY MULT. FOR A TWO-BYTE INTEGER
* MULTI. BY A SINGLE BYTE INTEGER
* MULTIPLIER IS IN OPR1+1
* MULTIPLICAND IS IN OPR2,OPR2+1
* RESULT WILL BE IN RSLT,RSLT+1
* (2 BYTES ARE DISCARDED)
*****
**** BEGIN SIMROUTINE ****
SMPY  PPSL     WC  SET MODE
      ENRZ     RN
      STRA,R0  RSLT  CLEAR RESULT
      STRA,R0  RSLT+1  CLEAR RESULT+1
      STRA,R0  OPR1
      LODI,R3  8  LOAD COUNT
      LUC0  LODA,R2  OPR1+1  GET MULTIPLIFR
      LUC0  RNR,R2  RUT. RIGHT WITH CARRY
* THIS ROTATES MULTIPLIER BY 1 BIT TO GET LSB INTO CARRY
      FORZ     RN  CLEAR RN
      RPL,R0  GET CARRY INTO 1SH
      RNR,R0  BRANCH IF C=0
      LODI,R1  2  INDEX
      LUC2  LODA,R0  RSLT-1,R1  ADD MULTIPLICAND TO PRODUCT
      ADDA,R0  OPR2-1,R1
      STRA,R0  RSLT-1,R1
      RNR,R1  LNC2  FINISH THE ADD

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LINE	ADDR	FILE	BI	92	93	94	ERRON	SOURCE
290	0889	0889	05	FE				LOC4 LOD1,R1 ~2 ROT. THE PRODUCT
291	0888	0888	00	63	79			LOC5 LODA,R0 RSLT-256+2,R1
292	088E		50					RRR,R0
293	088F		00	63	79			STRA,R0 RSLT-256+2,R1
294	0892		09	77				WRR,R1 LACS
295	0894		FB	61				WRR,R3 L000 FINISH THE LOOP
296								*****
297	0896		17					RETC,UN
298								*****
299								* A SUBROUTINE TO OUTPUT PITCH & ROLL DATA *
300								* CALLED BY MAIN PROG. ; FETCHES CONVERTED DATA *
301								* FROM TABLE "DATM" THEN O/P'S BCD ANGLE + SIGN *
302								*****
303	0897	0897	0C	04	00			OUTA LODA,R0 DAT+6 FETCH THETA (NSR)
304	089A		C3					STRZ R3
305	089B		44	00				ANDI,R0 H180° SAVE SIGN
306	089D		1A	10				BCTR,N NTH BR. ON NEG THETA
307	089F		2C	04	62			EORA,R0 DAT+8 COMPARE SIGNS
308	08A2		1A	06				BCTR,N NPH1
309	08A4		04	07				LODI,R0 7
310	08A6		04	10				WRITE,R0 SRTT BOTH POSITIVE
311	08A8		1B	14				BCTR,UN AGL
312	08AA	08AA	20					NPH1 EORZ R3
313	08AB		04	10				WRITE,R0 SRTT PHI NEG.
314	08AD		1B	0F				BCTR,UN AGL
315	08AF	08AF	2C	04	62			NTH EORA,R0 DAT+8
316	08B7		19	06				BCTR,P NPH
317	08B4		04	01				LODI,R0 3
318	08B6		04	10				WRITE,R0 SRTT PITCH NEG. & ROLL POS.
319	08B8		1B	0F				BCTR,UN AGL
320	08BA	08BA	04	08				NPH LODI,R0 8
321	08BC		04	10				WRITE,R0 SRTT BOTH NEGATIVE
322	08BE	08BE	7F	04	CC			AGL BSTA,UN L00 FORM ANGLE (BCD)
323	08C1		05	04				WRITE,R1 1HTA OUTPUT PITCH
324	08C3		0F	04	62			LODA,R3 DAT+8 FETCH ROLL
325	08C6		7F	04	CC			BSTA,UN L00 ANGLE (BCD)
326	08C9		05	04				WRITE,R1 PHI OUTPUT PHI
327	08CB		17					RETC,UN
328								*****
329	08CD	08CD	47	7F				LOD ANDI,R3 H17F°
330	08CE		CF	04	7C			STRA,R3 OPR2+1 MULTIPLICAND
331	08D1		20					EORZ R3
332	08D2		CC	04	74			STRA,R0 OPR2 CLEAR L5 BYTE
333	08D5		04	5A				LODI,R0 H15A° FACTOR (.35156)
334	08D7		CC	0F	7A			STRA,R0 OPR1+1 MULTIPLIER
335	08DA		3F	04	66			BSTA,UN SNPY BINARY ANGLE
336	08DD		0C	04	78			LODA,R0 RSL1+1 ANGLE
337	08F0		3F	0C	64			BSTA,UN BCD4 ANGLE (BCD)
338	08E1		25	FF				EORZ,R1 H1FF°
339	08E5		17					RETC,UN
340								*****

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LINE ADDR LABEL R1 R2 R3 R4 ERROR SOURCE

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342 *****
343 * S.M. ROTX TO CORRECT HX DATA USING ALGORITHM *
344 * HX=RX* COS(PITCH)+ SIN(PITCH)* (HYH* SIN(
345 * HULL)+ HZM* COS(HULL)) *
346 * ENTER WITH "RAW DATA" IN TABLE DATH *
347 * EXIT WITH "O)IFIED HX IN "DATH" *
348 * CALLS S.M. H7H1 *
349 * CALLS S.M.S. "SADD" & "SMPY" TO DO *
350 * SIGNED MAGNITUDE AND/SUBT AND MULTIPLY *
351 *****
352 08L6 08F6 77 00 ROTX PHSI WC+C AMTH WITH CARRY, C=1
353 08E8 0C 04 00 LODA,R0 DATH+0 THETA (PITCH)
354 08E8 C1 STRZ R1
355 08EC C1 STRZ R3
356 08E0 45 7F ANDI,R1 H17F+ SIRIP OFF SIGN
357 08LF 47 00 ANDI,R3 H180+ SAVE SIGN
358 08F1 CF 04 07 STRA,R3 SIN0 SIGN THETA
359 08F4 00 65 00 LODA,R0 SIN,R1 SIN(THETA)
360 08F7 CC 04 08 STRA,R0 SIN0+1
361 08FA 00 65 09 LODA,R0 COS,R1 COS(THETA)
362 08FD CC 04 05 STRA,R0 COST
363 0900 CC 04 7A STRA,R0 OPR1+1 MULTIPLIER
364 0903 0C 04 5A LODA,R0 DATH HX MSBITS
365 0906 C1 STRZ R1
366 0907 44 7F ANDI,R0 H17F+ SIRIP OFF SIGN
367 0909 45 09 ANDI,R1 H180+ SAVE SIGN
368 090A CC 04 7B STRA,R0 OPR2 MULTIPLICAND
369 090E CJ 04 06 STRA,R1 SIGA
370 0911 JC 04 5B LODA,R0 DATH+1
371 0914 CC 04 7C STRA,R0 OPR2+1
372 0917 IF 0B 66 BSTA,UN SMPY FORM HX* COS (THETA)
373 091A 0C 04 77 LODA,R0 RSLT MOVE PRODUCT
374 091D CC 04 0B STRA,R0 CRUF
375 0920 0C 04 7B LODA,R0 RSLT+1
376 0923 CC 04 0C STRA,R0 D1UF
377 0926 JF 0F 40 BSTA,UN H/R1
378 0929 0C 04 08 LODA,R0 SIN0+1 SIN (THETA)
379 092C CC 04 7A STRA,R0 OPR1+1 MULTIPLIER
380 092F 3F 0B 66 BSTA,UN SMPY HZ* SIN(THETA)
381 0932 0C 04 67 LODA,R0 SIN0 SIGN OF THETA
382 0935 CC 04 7E STRA,R0 SEES FOR USE BY SADD
383 0938 00 04 06 LODA,R1 SIGA
384 093B CC 04 7D STRA,R1 STE1
385 093E 0C 04 0B LODA,R0 CRUF
386 0941 00 04 0C LODA,R1 D1UF
387 0944 CC 04 0B STRA,R0 TFMH
388 0947 00 04 06 STRA,R1 TFMH+1
389 094A 75 07 CPSI CON FLAG = AD0
390 094C IF 04 0B HST1,UN SADD FORM NEW HX
391 094F 0C 04 7B LODA,R0 RSLT+1 MOVE NEW HX
392 0952 CC 04 0D STRA,R0 DATH+1
393 0955 0C 04 77 LODA,R0 RSLT

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LINE	ADDR	LABL	B1	B2	B3	B4	ERROR	SOURCE
394	0958		CC	04	7F			STRA,R0 DATA
395	0958		17					RETC,UN
396								*****
397								* SUBROUTINE TO SQUARE TWO 2 BYTE VALUES *
398								* ENTER WITH VALUE IN R1(MSB) & R2(LSB) *
399								* EXIT WITH 2 MSD IN RSLT & RSLT+1 *
400								*****
401								**** BEGIN ****
402	095C	095C	77	0A				SOU PPSL R0
403	095E		70					EORL R0
404	095F		LC	04	77			STRA,R0 RSLT
405	0962		CC	04	7A			STRA,R0 RSLT+1 CLEAR TEMP STORAGE
406	0965		07	10				LUDI,R3 16 COUNTER
407	0967		45	7F				ANDI,R1 H:7F
408	0969		CD	04	7B			STRA,R1 OPR2
409	096C		CE	04	7C			STRA,R2 OPR2+1
410	096F	096F	S1					ROTA RRR,R1 ROTATE MULTIPLIER
411	0970		52					RRR,R2
412	0971		20					EORL R0
413	0972		00					RRL,R0
414	0973		F8	12				BDRH,R0 NOAD
415	0975		0C	04	7B			LODA,R0 RSLT+1
416	0978		BC	04	7C			ADDA,R0 OPR2+1
417	097B		CC	04	7A			STRA,R0 RSLT+1
418	097E		0C	04	77			LODA,R0 RSLT
419	0981		8C	04	7B			ADDA,R0 OPR2
420	0984		1F	04	7A			BCTH,UN NOAD+3
421	0987	0987	0C	04	77			NOAD LODA,R0 RSLT
422	098A		50					RRR,R0
423	098D		CC	04	77			STRA,R0 RSLT
424	098F		0C	04	7B			LODA,R0 RSLT+1
425	0991		50					RRR,R0
426	0992		CC	04	7B			STRA,R0 RSLT+1
427	0995		F8	5A				BDRH,R3 R071
428								*****
429	0997		17					CONT. MULT. IF DEF.
								*****
								RTCL,UN
								END OF SUBROUTINE

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LINE ADDR LABEL R1 R2 R3 R4 FROM SOURCE

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431 *****
432 * S.R. SADD TO PERFORM DOUBLE PRECISION *
433 * SIGN & MAGNITUDE ADD/SUBT *
434 * ADDEND (SUBT-ADDEND) IN TEMP, TEMP+1 *
435 * ADDEND (MAGNITUDE) IN RSLT, RSLT+1 *
436 * STEM = SIGN OF TERM IN TEMP SRFS = SIG *
437 * CUM BIT IS FLAG ((0=ADD) 1= SUBT) *
438 * SIGNED TOTAL LEFT IN RSLT, RSLT+1 *
439 *****
440 0998 0998 77 04 SADD PPSL WC+C ARITH WITH CARRY; SFT CARRY
441 099A 07 02 LOUI,R3 2 INDEX
442 099C 45 02 TPST CUM TEST FLAG
443 099E 1C 0A 1A BCTA,UN SSUM HK, IF SUMT.
444 * START ADD HERE
445 09A1 0F 0A 7D LOUA,R3 STEM SIGN OF TEMP
446 09A4 13 00 LODZ R3 EX. OR WITH SIGN RESULT
447 09A5 2C 0A 7E EORA,R0 SRFS
448 09A8 1E 09 CA BCTA,N DIFFR BK, IF SIGNS DIFFR
449 09AB 75 01 CPST C
450 09AD 0C 0A 66 LOUA,R0 TEMP+1 FORM SUM
451 09B0 4C 0A 78 ADDA,R0 RSLT+1
452 09B3 0C 0A 78 STRA,R0 RSLT+1
453 09B6 0C 0A 65 LOUA,R0 TEMP
454 09B9 8C 0A 77 ADDA,R0 RSLT
455 09BC 9C 09 F9 BCTA,Z KTR,
456 09BF 0D 0A 78 LOUA,R1 RSLT+1 GET LS BITS
457 09C2 9C 09 F9 BCTA,Z KTR,
458 09C5 20 00 EORZ R0
459 09C6 0C 0A 77 STRA,R0 RSLT ZERO RSLT = POS
460 09C9 17 00 KETC,UN
461 09CA 06 02 01FR LOUI,R2 2 CIR/INDEX
462 09CC 03 00 LODZ R3
463 09CD 1E 0A 02 BCTA,1 X REG BK, IF MX = NEG
464 09D0 0E 0A 04 SUP1 LOUA,R0 TEMP-1,R2 FORM TEMP - RSLT
465 09D3 8E 0A 76 SUBA,R0 RSLT-1,R2
466 09D6 0E 0A 76 STRA,R0 RSLT-1,R2
467 09D9 FA 75 BURN,R2 SUP1
468 09DB 45 01 TPST C
469 09DD 1C 0A 12 BCTA,UN 17EX HK, IF C = 1
470 09F0 77 01 71FR PPSL C
471 09E2 0B 02 SUP2 LOUI,R2 2 INDEX/CIR
472 09E4 09F4 20 00 EORZ R0 FORM 2'S COMPLEMENT
473 09E5 8E 0A 76 SUBA,R0 RSLT-1,R2 IF C = 0
474 09E8 0E 0A 76 STRA,R0 RSLT-1,R2
475 09EB FA 77 BURN,R2 SUP2
476 09ED 01 00 STRZ R1
477 09EE 9C 09 FE BCTA,Z KTR,
478 09F1 0D 0A 78 LOUA,R1 RSLT+1
479 09F4 9C 09 FE BCTA,Z KTR,
480 09F7 1B 01 BCTA,UN KTR,
481 09F9 03 00 KTRN LOUA,R3
482 09FA 0C 0A 77 KTRN STRA,R0 RSLT

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PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMBLY PROGRAM 1976 PAGE 1

LINE	ADDR	INSTR	H1	H2	H3	H4	ERROR	SOURCE
483	09FD		17					RETC,UN
484	09FE	09FE	64	80				WRES INRI,R0 H*80*
485	0A00		1B	7A				BCFI,UN NOW
486								* START HERE IF TERM IN "TEMP" IS NEGATIVE
487	0A02	0A02	0E	64	76			XNEG LODA,R0 RSLT-1,R2 FORM DIFFERENCE
488	0A05		AE	64	64			SUBA,R0 TEMP-1,R2
489	0A08		CE	64	76			STRA,R0 RSLT-1,R2
490	0A0B		FA	75				BDRH,R2 XNEG LOOP UNCE
491	0A0D		85	01				TPSL C
492	0A0F		9C	09	F0			BCFA,UN INEG
493	0A12	0A12	44	7F				YZER ANDI,R0 H*7F* RESULT IS POS
494	0A14		CC	04	77			STRA,R0 RSLT
495	0A17		17					RETC,UN
496								*****
497								* START HERE IF SUBTRACTION IS REQUIRED
498								* I.E. (RSLT,RSLT*1) - (TEMP,TEMP*1)
499	0A18	0A18	0C	04	7E			SSUB LODA,R0 SRES SIGN OF RESULT
500	0A1B		C3					STR7 R3
501	0A1C		06	02				LOUI,R2 2
502	0A1E		2C	04	7D			EDRA,R0 STEM
503	0A21		4E	0A	36			BCFA,N SUBH BN, IF SIGNS SAME
504	0A24		75	01				CPSL C
505	0A26	0A26	0E	64	64			SLUP LODA,R0 TEMP-1,R2
506	0A29		AE	64	76			ADDA,R0 RSLT-1,R2
507	0A2C		CE	64	76			STRA,R0 RSLT-1,R2
508	0A2F		FA	75				BDRH,R2 SLUP
509	0A31		53					INRZ R3
510	0A32		CC	04	77			STRA,R0 RSLT
511	0A35		17					RETC,UN

LINE	ADDR	LABL	H1	H2	H3	H4	ERROR	SOURCE
513								* START HERE IF BOTH TERMS SAME SIGN
514	0A36	0A36	77	61				SUBR PPSL C C=1
515	0A38	0A38	0E	64	76			HLUP LODA,R0 RSLT-1,R2 RSLT = TEMP - RSLT
516	0A38		0E	64	76			SUBA,R0 TEMP-1,R2
517	0A3E		CE	74	76			STRA,R0 RSLT-1,P2
518	0A41		FA	75				BORR,R2 B1 UP
519	0A43		05	01				TPSL C TEST CARRY
520	0A45		1C	0A	5F			BCTA,ON FINS
521	0A4B		06	02				LODI,R2 2
522	0A4A		77	01				PPSL C
523	0A4C	0A4C	20					CLUP EORZ R0 LOOP TO DO
524	0A4D		0E	64	76			SUBA,R0 RSLT-1,R2 2'S COMPL.
525	0A50		CE	64	76			STRA,R0 RSLT-1,R2
526	0A53		FA	77				BORR,R2 C1 UP
527	0A55		06	04	7E			LODA,R1 S4E5
528	0A5A		16					RETC,N TEST SIGN
529	0A59		04	00				LODI,R0 H1B0'
530	0A5H		CL	04	77			STRA,R0 RSLT
531	0A5E		17					RETC,ON
532	0A5F	0A5F	01					FINS STRZ W1
533	0A60		9C	0A	6A			BCFA,7 FAN
534	0A67		00	04	78			LODA,R1 RSLT+1
535	0A66		9C	0A	6A			BCFA,2 FAN
536	0A69		17					RETC,ON
537	0A6A	0A6A	67					FAN LORZ R3 SIGN = STEH
538	0A6B		CL	04	77			STRA,R0 RSLT
539	0A6E		77	02				PPSL C10M
540	0A70		17					RETC,ON
541								*****

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LINE ADDR LABEL B1 B2 B3 B4 FROM SOURCE

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543
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545
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549
550
551 0A71 0A71 77 04
552 0A73 0C 04 42
553 0A76 C1
554 0A77 45 7F
555 0A79 2C 04 5E
556 0A7C 44 80
557 0A7E 0A7E CC 04 7D
558 0A81 0D 64 79
559 0A84 CC 04 75
560 0A87 0D 64 70
561 0A8A 7C 04 7A
562 0A8D 0C 04 5E
563 0A90 44 7F
564 0A92 CC 04 7B
565 0A95 0C 04 5F
566 0A98 CC 04 7C
567 0A9B 7F 64 66
568 0A9E 0C 04 77
569 0AA1 CC 04 65
570 0AA4 0C 04 7B
571 0AA7 CC 04 66
572
573 0AAA 0C 04 5C
574 0AA0 C1
575 0AAE 45 7F
576 0AB0 CD 04 76
577 0AB3 44 80
578 0AB5 7C 04 7F
579 0AB8 0C 04 5D
580 0ABA CC 04 7C
581 0ABE 0C 04 75
582 0AC1 7C 04 7A
583 0AC4 3F 04 06
584
585 0AC7 77 02
586 0AC9 7F 02 76
587 0ACC 0C 04 77
588 0ACF CC 04 71
589 0AD2 0C 04 7A
590 0AD5 CC 04 72
591 0AD8 77

*****
* S.H. ROTY TO ADJUST HY DATA USING ALGORITHM *
* HY=HY+*COS(ROLL)-*H*Z*M*SIN(ROLL) *
* USES SIGN AND MAGNITUDE DATA FROM TABLE DATM *
* STORES NEW DATA INTO DATY, DATY+1 *
* CALLS S.H.S. "SADD" & "SMPY" TO PERFORM SIGNED *
* MAGNITUDE ARITHMETIC *
*****
ROTY PPSL WC+C
      LODA,R0 DATM+H GET ROLL ANGLE PHI
      STRZ R1
      ANDI,R1 H:7F STRIP OFF SIGN
      EORA,R0 DATM+4 DETERMINE SIGN OF PROD.
      ANDI,R0 H:80
      STRA,R0 STEM
      LODA,R0 COS,R1 COS (PHI)
      STRA,R0 COSH
      LODA,R0 SIN,R1 SIN (PHI)
      STRA,R0 OPR1+1 MULTIPLIER
      LODA,R0 DATM+4 GET HZ
      ANDI,R0 H:7F STRIP SIGN
      STRA,R0 OPR2 MULTIPLICAND
      LODA,R0 DATM+5
      STRA,R0 OPR2+1
      BSTA,UN SMPY FORM 22 SIN(PHI)
      LODA,R0 RSLT MOVE PRODUCT
      STRA,R0 YEMP
      LODA,R0 RSLT+1
      STRA,R0 YEMP+1
* BEGIN FORMING 2ND TERM
      LODA,R0 DAT+2 GET HY 45 BITS
      STRZ R1
      ANDI,R1 H:7F STRIP SIGN
      STRA,R1 OPR2 MULTIPLICAND
      ANDI,R0 H:80 SAVE SIGN
      STRA,R0 SRES SIGN OF RESULT
      LODA,R0 DATM+3 HY LS BITS
      STRA,R0 OPR2+1
      LODA,R0 COSH
      STRA,R0 OPR1+1 MULTIPLIER
      BSTA,UN SMPY
* NOW FORM NEW HY
      PPSL COM
      BSTA,UN SADD
      LODA,R0 RSLT MOVE ROTATED HY
      STRA,R0 DATY
      LODA,R0 RSLT+1
      STRA,R0 DATY+1
      RETC,UN

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REPRODUCIBILITY OF THE  
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LINE	ADDR	INSTR	HL	SP	BP	PC	SOURCE
593							*****
594							* S.R. HVEC TO COMPUTE HORIZ. FIELD VECTOR
595							* HX = (HUK1/2)*2 + HY (HUK1/2)*2
596							* GETS HX & Y FROM (DATA,DATA+1) & (DATA+2,DATA+3)
597							* RESULT IN (HUK1, HUK1+1)
598							*****
599	0A09	0A09	77	08			HVEC PUSL WC
600	0A0A		00	04	7F		LOAD, H1 DATA GET ROTATED HX
601	0A0E		0E	04	7F		LOAD, H2 DATA+1
602	0A11		3F	04	5C		STRA, H0 SQUARE HX
603	0A14		0C	04	77		LOAD, H0 RSL1 MOVE RESULT
604	0A17		CC	04	7F		STRA, H0 HX2
605	0A1A		CC	04	7B		LOAD, H0 RSL1+1
606	0A1D		CC	04	7F		STRA, H0 HX2+1
607	0A20		0D	04	71		LOAD, H1 DATA GET HY (ROTATED)
608	0A23		0E	04	72		LOAD, H2 DATA+1
609	0A26		3F	04	5C		STRA, H0 SQUARE HY
610	0A29		0B	04	77		LOAD, H2 RSL1-1, R2
611	0A2C	0AFB	0E	04	7B		STRA, H0 HY2-1, R2 MOVE HY2
612	0A2F		CE	04	7F		STRA, H0 HY2-1, R2
613	0A32		FE	04	7F		STRA, H2 LUPF
614	0A35		7D	04	71		CMPL C
615	0A38		0B	04	77		LOAD, H2 RSL1-1, R2
616	0A3B	0A0B	0E	04	7B		LOAD, H0 HX2-1, R2 SUM HX12+HY12
617	0A3E		BE	04	7B		STRA, H0 RSL1-1, R2
618	0A41		CE	04	7A		STRA, H0 HX2-1, R2
619	0A44		FA	04	7C		STRA, H2 LUPF
620	0A47		17				RETC, H0
621							*****
622							* S.R. TO COMPUTE YAW = ANGLE FROM THE HUK1/2
623							* VECTOR COMP. H0
624							* ENTER WHEN MAGN(HX) < MAGN(HY)
625							* YAW = (HUK1/2)/HUK1/2
626							* HUK1 = DIVIS HUK1... = HUK1... = DIVIDEND
627							* RSL1 = QUOTIENT
628							*****
629							*****
630	0B14	0B14	0B	04	77		COSY LOAD, H1 2 INDEX
631	0B17	0B17	0D	04	7B		LOAD, H0 HX2-1, R1 LOAD (HUK1-1)/HUK1
632	0B1A		0D	04	77		STRA, H0 (HUK1-1)/HUK1
633	0B1D		74	04	7B		STRA, H1 LUPF LOOP ONCE
634	0B20		3F	04	7A		STRA, H0 DIV1 FORM HX2/HY2
635	0B23		3F	04	7B		STRA, H0 ANGUL FIND ARCUS
636	0B26		3F	04	7A		STRA, H0 HCUA CONVERT TO HCU
637	0B29		3F	04	7C		STRA, H0 HCU COMPUTE HEADING
638	0B2C		17				RETC, H0
639							*****

LINE	ADDR	LABEL	B1	B2	B3	B4	FROM	SOURCE
641								*****
642								* A SUBROUTINE TO COMPUTE HEADING WHEN *
643								* ( +/- ) 135 < YAW < ( +/- ) 45 DEGREES *
644								*****
645	0B2B	0B2B	05	02				SINY LODI,R1 2 INDEX
646	0B2D	0B2D	00	64	6F			LUP LODA,R0 HY2-1,R1 LOAD OPR1-1,OPR1
647	0B30		CD	64	7B			STRA,R0 OPR1-1,R1
648	0B33		F9	7A				BDRR,R1 LOP
649	0B35		20					EORZ R0
650	0B36		7F	0A	4A			BSTA,UN UIV1 FURN HY2/HH2
651	0B39		3F	0A	4B			BSTA,UN ANGL FURN ARCOS(HY2/HH2)
652	0B3C		3F	0C	64			BSTA,UN BCDA CONVERT TO HCH
653	0B3F		04	90				LODI,R0 H+90 CALC. 90-ANGLF
654	0B41		77	04				PPSL C+WC
655	0B43		A1					SUHZ K1
656	0B44		94					DAR,R0
657	0B45		C1					STRZ R1
658	0B46		7F	0C	7C			BSTA,UN HNG
659	0B49		17					RETC,UN
660								*****



LINE	ADDR	LABEL	B1	B2	B3	B4	EXPLAN	SOURCE
602								*****
603								* A SUBROUTINE TO DIVIDE HX2/HM2 OR HY2/HM2 *
604								* ENTER WITH HX2 OR HY2 IN OPR1 & OPR1+1 *
605								* EXIT WITH QUOTIENT IN RSLT,RSLT+1 *
606								*****
607	0B4A	0B4A	20					DIV: EORZ R0
608	0B4B		CC	04	77			STRA,R0 RSLT CLR QUOTIENT
609	0B4E		CC	04	78			STRA,R0 RSLT+1
610	0B51		07	11				LOUL,R3 17 CLR
611	0B53		0E	04	AC			LODA,R2 HM2+1
612	0B56		00	04	0B			LODA,R1 HMC
613	0B59	0B59	77	04				PPSL CWM+CWC LOGICAL COMP..C=1, WITH C
614	0B5B		E5	00				COM1,R1 0
615	0B5D		1C	03	53			BCTA,Z ZFR
616	0B60		FD	04	74			COMA,R1 OPR1
617	0B63		14	10				BCTH,B1 GTD BR. IF HM2>HX+2
618	0B65		1A	05				BCTH,LT GTD BR. IF HM2<HX+2
619	0B67		EE	04	7A			COMA,R2 OPR1+1
620	0B6A		19	12				BCTH,GT GTD BR. IF HM>HA (LS BITS)
621								**** HERE IF HM<HX OR HM=HX (ALL BITS)
622	0B6C	0B6C	0C	04	7A			LODA,R0 OPR1+1
623	0B6F		A2					SUBZ R2 FORM DIVIDEND-DIVISOR
624	0B70		CC	04	7A			STRA,R0 OPR1+1
625	0B73		0C	04	74			LODA,R0 OPR1
626	0B76		A1					SUBZ R1
627	0B77		CC	04	74			STRA,R0 OPR1
628	0B7A		77	01				PPSL (
629	0B7C		10	02				BCTH,UN JMP
630	0B7E	0B7E	75	01				PPSL C
631	0B80	0B80	0C	04	78			LODA,R0 RSLT+1
632	0B83		00					RRL,R0 LSH=1 OR 0
633	0B84		CC	04	78			STRA,R0 RSLT+1 STR. QUOTIENT (C=0)
634	0B87		0C	04	77			LODA,R0 RSLT
635	0B8A		00					RRL,R0 LSH=1 OR 0
636	0B8B		CC	04	77			STRA,R0 RSLT STR. QUOTIENT (C=0)
637	0B8F		51					RRL,R1 ROT. DIVISOR WITH C
638	0B8F		52					RRL,R2
639	0B90		EA	47				BORR,R3 LMP1 CONT. UNTIL DONE
700								**** HERE PSI T=CONS(YAW1)
701	0B92		17					BCTC,UN
702	0B93	0B93	FD	00				COM1,R2 0
703	0B95		9C	04	40			BCTA,Z LMP1+1
704	0B98		1F	04	7E			BCTA,UN GTD
705								*****
706								*****
707								* A SUBROUTINE TO CALCULATE THE ARCUS FUNCTION *
708								* THE ALGORITHM USES SUCCESSIVE APPROXIMATION *
709								* & TABLE LOOK-UP TO ITERATE TO THE SOLUTION *
710								* BEGIN WITH ANGLE IN RSLT,RSLT+1 *
711								* EXIT WITH ARCOS(ANG) IN R0 *
712								*****
713	0B9B	0B9B	77	04				ANGL PPSL CWC+CWM

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LINE	ADDR	LA-L	B1	B2	B3	B4	FROM	SOURCE
714	0B9D		0E	04	77			LODA,R2 RSLT MS BYTE OF ARG.
715	0BA0		05	17				LODI,R1 23 1ST ESTIMATE
716	0BA2		07	17				LODI,R3 23 TWICE 1ST INCREMENT
717	0BA4	0BA4	0D	6A	HA		LUPA	LODA,R0 COSM,R1 COS(EST.)
718	0BA7		75	01				CPSL C MODIFY & TEST CTR.
719	0BA9		5J					RHR,R3
720	0BAA		1B	1F				BCTR,Z LAST
721	0BAC		E2					CONZ R2
722	0BAD		19	0D				BCTR,GT ELTA
723	0BAF		1A	12				BCTR,LT EGTA
724	0BB1	0BB1	E4	01			EQL1	CON1,R0 1
725	0BB3		1D	0C	43			BCTA,GT QUIT
726	0BB6		1C	0C	49			BCTA,E0 ONE
727	0BB9		1F	0C	27			BCTA,UN ITLS MS BITS ARE EQU. & < 1
728	0BBC	0BBC	75	01			ELTA	CPSL C INCR. EST.
729	0BBE		01					LOU2 R1
730	0BBF		03					ADD2 R3
731	0BC0		C1					STRZ R1
732	0BC1		1B	01				BCTR,UN LUPA
733	0BC3	0BC3	77	01			EGTA	PPSL C CONTINUE LOOPING
734	0BC5		01					LOU2 R1 DECR. EST.
735	0BC6		A3					SUB2 R3
736	0BC7		C1					STRZ R1
737	0BCA		1B	5A				BCTR,UN LUPA
738	0BCA	0BCA	07	0A			LAST	LODI,R3 10
739	0BCC	0BCC	0D	6A	HA		LUPB	LODA,R0 COSM,R1
740	0BCF		F2					CONZ R2
741	0BD0		19	05				BCTR,GT E1 T
742	0BD2		1A	09				BCTR,LT EGT
743	0BD4		1F	08	B1			BCTA,UN EQL1
744	0BD7	0BD7	75	01			ELT	CPSL C
745	0BD9		05	01				ADD1,R1 1
746	0BDH		1A	04				BCTR,UN ISTR
747	0BD0	0BD0	77	01			EGT	PPSL C
748	0BDF		A5	01				SUB1,R1 1
749	0BE1	0BE1	FB	6A			ISTR	BDRR,R3 LUPB
750								LOOP IF R3=0
751	0BE3		0D	6A	HA			LODA,R0 COSM,R1
752	0BE6		E2					CONZ R2
753	0BE7		19	1A				BCTR,GT IGT
754								START IF COS(EST.) < ARG.
755	0BE9		C3					STRZ R3
756	0BEA		0D	6A	19			LODA,R0 COSM-1,R1
757	0BED		77	01				PPSL C
758	0BEF		A3					SUB2 R3
759	0BF0		75	01				CPSL C
760	0BF2		50					RHR,R0
761	0BF3		75	01				CPSL C
762	0BF5		03					ADD2 R3
763	0BF6		F2					CONZ R2
764	0BF7		1E	0C	1			BCTR,LT
765	0BFA		1F	01				PPSL C

REPRODUCIBILITY OF THE  
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LINE	ADDR	LAB	H1	B2	H3	B4	FROM	SOURCE
766	0BFC		45	01				SUM1,R1 1
767	0BFF		1F	0C	43			BCTA,UN UNIT
768								* START HERE IF COS(PST.) > ARG.
769	0C01	0C01	F4	02				IGT CUM1,R0 2
770	0C03		9H	07				BCTH,EO NEQ? RM. IF COS(ESI) > ?
771	0C05		75	01				CPSL C
772	0C07		45	01				ADUI,R1 1
773	0C09		1F	0C	43			BCTH,UN UNIT
774	0C0C	1C C	00	6A	7A			NEQ2 LODA,R0 COS+1,R1
775	0C0F		C1					STRZ R3
776	0C10		00	6A	7A			LODA,R0 COS+1,R1
777	0C13		77	01				PPSL C
778	0C15		A3					SUBZ R3
779	0C16		75	01				CPSL C
780	0C18		50					RRR,R0
781	0C19		75	01				CPSL C
782	0C1B		03					ADDZ R3
783	0C1C		E2					CUMZ R2
784	0C1D		1E	0C	43			BCTA,LT UNIT
785	0C20		75	01				CPSL C
786	0C22		F5	01				ADUI,P1 1
787	0C24		1F	0C	43			BCTA,UN UNIT
788								* EXAMINE LS BITS
789	0C27	0C27	75	01				ITLS CPSL C BEGIN ITERATION ON LS BITS
790	0C29		05	2A				LODI,R1 42 ESTIMATE OF ADDRESS
791	0C2H		0E	04	7B			LODA,R2 RSLT+1
792	0C2F		F6	71				CUM1,R2 H+H1 1ST ESTIMATE
793	0C30		10	0C	43			BCTA,GT UNIT
794	0C33		05	01				ADUI,R1 1
795	0C35		F6	31				CUM1,R2 H+J1 2ND EST.
796	0C37		10	0C	43			BCTA,GT UNIT
797	0C3A		05	01				ADUI,R1 1
798	0C3C		F6	09				CUM1,R2 H+09 3D EST.
799	0C3F		10	0C	43			BCTA,GT UNIT
800	0C41		45	01				ADUI,R1 1
801	0C43	0C43	01					UNIT LODZ R1
802	0C44		75	01				CPSL C
803	0C46		04	71				ADUI,R0 45
804								*****
805	0C4H		17					RFIC,UN
806	0C49	0C49	05	2A				ONE LODI,R1 40
807	0C4H		00	6A	7A			LODA,R0 COSL+1,R1
808	0C4F		C3					STRZ R3
809	0C4F		00	6A	7A			LODA,R0 COSL+1,R1
810	0C52		77	01				PPSL C
811	0C54		A3					SUBZ R3
812	0C55		75	01				CPSL C
813	0C57		50					RRR,R0
814	0C58		75	01				CPSL C
815	0C5A		R3					ADDZ R3
816	0C5B		FC	04	7B			COMA,R0 RSLT+1
817	0C5F		1A	61				BCTH,LT UNIT

REPRODUCIBILITY OF THE  
ORIGINAL PAGE IS POOR

LINE	ADDR	LAHL	B1	B2	B3	B4	ERROR	SOURCE
818	0C60		05	29				LODI,R1 4}
819	0C62		18	5F				BCTR,UN 0011
820								*****
821								*****
822								" A SUBROUTINE TO CONVERT BINARY ANGLE TO BCD "
823								* ENTR WITH R0=ANGLE(DIN) & EXIT R1=BCD EQUIV. *
824								*****
825	0C64	0C64	07	07				HCD4 LODI,R3 7 CTR.
826	0C66		05	00				LODI,R1 0 CLR TOTAL
827	0C68	0C68	75	01				ARCH CPSL C CLR. C
828	0C6A		00					RRL,R0
829	0C6B		C2					STRZ R2 MOVE RESULT
830	0C6C		9A	0A				BCFH,N NJNC BR. IF MS BIT=0
831	0C6E		01					LODZ R1
832	0C6F		75	01				CPSL C
833	0C71		84	66				ADDI,R0 H+66+ BCD ADD 2 BYTES
834	0C73		8F	67	15			ADDA,R0 BCD-1,R3 INCR. BCD TOTAL
835	0C76		94					DAR,R0
836	0C77		C1					STRZ R1
837	0C7A		02					LODZ R2 RESULT TO R0
838	0C79	0C79	F0	60				NINC BDRH,R3 BRCH GO BACK TIL DONE
839								**** LEAVE WITH BCD EQUIV. OF YAW1 IN R1
840	0C7B		17					RETC,UN
841								*****

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LINE ADDR LABEL H1 H2 H3 H4 FROM SOURCE

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841
844
845
846
847 0C7C 0C7C 77 04
848 0C7E 0C 04 IF
849 0C71 1A 1R
850
851 0C73 0C 04 H1
852 0C76 1A 1R
853
854
855 0C88 75 00
856 0C8A 18 0C
857 0C8C 04 50
858 0C8E 1A 1R
859 0C8F 04
860 0C90 03
861 0C91 06 01
862 0C93 16 00
863 0C95 06
864 0C96 15 27
865
866
867 0C9A 0C9A 0A 00
868 0C9A 01
869 0C9B 03
870 0C9C 1A 21
871
872
873 0C9E 0C9E 0C 04 H1
874 0CA1 1A 1R

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*****
* A SUBROUTINE TO CALCULATE HEADING & OUTPUT *
* THREE H10 DIGITS PLUS SIGN *
*****
H10  PPSL  H10C
      LODA,R0  DATA  SIGN OF HX ?
      BCTR,M  YAW0  0H, IF NEG
* SOLVE FOR HEADING (HCD FORM)
      LODA,R0  DATA  GET HY
      BCTR,M  HY0  0H, ON HY=NEG,
*
* START HERE 0 POS, HY & HX
      COMI,K1  0
      BCTR,M  HY0
      LODI,K0  H1001
      SUBZ  H1
      DASH,R0
      STRZ  H2
      LODI,K2  H1001
      SUBI,K2  0
      DASH,R1
      BCTR,M  FINI
*
* ENTER HERE IF HX=POS. & HY=NEG,
HY0  LODI,K2  H1001
      LODZ  R1
      STRZ  R2
      BCTR,M  FINI
*
* ENTER HERE HX=NEG & HY= +/-
YAW0  LODA,R0  DATA  TEST SIGN OF HY
      BCTR,M  HY0

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LINE	ADDR	INSTR	H1	H2	H3	H4	ERROR	SOURCE
876								* ENTER HERE IF HX=NEG & HY=POS
877	0CA3		04	80				LODI,R0 H:801
878	0CA5		75	01				CPSL C
879	0CA7		84	66				ADDI,R0 H:601
880	0CA9		81					ADDZ R1
881	0CAA		94					DAR,R0
882	0CAB		C3					STRZ R3
883	0CAC		06	01				LODI,R2 H:011
884	0CAE		86	66				ADDI,R2 H:661
885	0CH0		86	00				ADDI,R2 0
886	0CH2		96					DAR,R2
887	0CH3		18	0A				BCTR,UN FINE
888								* ENTER HERE IF HX & HY = NEG
889	0CH5	0CH5	04	80				NHY LODI,R0 H:801
890	0CH7	0CH7	A1					DO SUBZ R1 SUB. YAW1 LSH
891	0CH8		94					DAR,R0
892	0CH9		C3					STRZ R3
893	0CHA		06	01				LODI,R2 H:011
894	0CHC		A6	00				SUBI,R2 0
895	0CHF		96					DAR,R2
896								*
897								*
898								**** READOUT OF DATA ****
899	0CHF	0CHF	26	FF				FINE EORI,R2 H:FF1
900	0CC1		06	01				WRITE,R2 MSB MSB OUT
901	0CC3		27	FF				EORI,R3 H:FF1
902	0CC5		07	02				WRITE,R3 LSH LSH OUT
903	0CL7		17					RETC,UN
904								*****
905								ORG H:401
906								*****
907								* S.N. TO CALC. THE HZ COMPONENT IN X-Z PLANE *
908								* HZ=HY*SIN(PHI) + HX*COS(PHI) *
909								* EXIT WITH HZ IN DRR2,DRR2+1 *
910								*****
911	0E40	0E40	00	04	62			HZR1 LODA,R1 DATM+8 GET PHI
912	0E43		0E	04	62			LODA,R2 DATM+8
913	0E46		45	80				ANDI,R1 H:801 SAVE SIGN
914	0E49		46	7F				ANDI,R2 H:7F1 STRI SIGN
915	0E4A		C0	04	80			STRA,R1 TRUE HOLD SIGN
916	0E4D		0E	65	00			LODA,R0 SIN,R2 SIN(PHI)
917	0E50		CC	04	69			STRA,R0 SIN
918	0E53		0E	65	89			LODA,R0 COS,R2 COS(PHI)
919	0E56		CC	04	7A			STRA,R0 DRR2+1
920	0E59		0C	04	7E			LODA,R0 DATM+4 HZ
921	0E5C		0D	04	7F			LODA,R1 DATM+5
922	0E5F		CC	04	7D			STRA,R0 DRR2
923	0E62		C0	04	7C			STRA,R1 DRR2+1
924	0E65		3F	04	66			BSTRA,UN SWP1 HZ*COS(PHI)
925	0E68		0C	04	77			LODA,R0 RSL1
926	0E6B		0D	04	78			LODA,R1 RSL1+1
927	0E6F		CC	04	89			STRA,R0 ANUF

LINE	ADDR	INSTR	B1	B2	B3	B4	FROM	SOURCE
928	0E71	CD	04	HA				STRA,R1 BRUF
929	0E74	0C	04	69				LOUA,R0 SINR
930	0E77	CC	04	7A				STRA,R0 OPR1+1
931	0E7A	0C	04	5C				LOUA,R0 UATM+2 HY
932	0E7D	0D	04	50				LOUA,R1 UATM+3
933	0E80	C2						STN2 R2
934	0E81	44	7F					ANDI,R0 H17F
935	0E83	46	80					ANDI,R2 H180
936	0E85	7E	04	AD				EORR,R2 TOUT DETERMINE SIGN
937	0E88	CC	04	7A				STRA,R0 UPR2
938	0E8B	CD	04	7C				STRA,R1 UPR2+1
939	0E8E	CE	04	7E				STRA,R2 SRES
940	0E91	3F	08	86				BSTA,UN SHPY HY=SIN(PH1)
941	0E94	0C	04	89				LOUA,R0 ARUF
942	0E97	1D	04	8A				LOUA,R1 BRUF
943	0E9A	CC	04	8B				STRA,R0 TFM2
944	0E9D	CD	04	8B				STRA,R1 TFM2+1
945	0EA0	7D	04					CRSL CDM AUD
946	0EA2	20						LOR2 R0
947	0EA3	CC	04	7D				STRA,R0 SYEM HZ=CDS(PH1)=PH5
948	0EA6	3F	08	8B				HSTA,UN SADD
949	0EA9	0C	04	77				LOUA,R0 KSLT
950	0EAC	0D	04	7D				LOUA,R1 KSLT+1
951	0EAF	CC	04	78				STRA,R0 UPR2
952	0EH2	CD	04	7C				STRA,R1 UPR2+1
953	0EH5	77	02					PPSL CDM
954	0EH7	17						REFL,UN

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LINE	ADDR	LAPL	B1	B2	B3	B4	ERROR	SOURCE
956								*****
957								* S.R. TO CORRECT NONORTHOGONALITY ERRORS IN HX *
958								* ENTER ONCE EACH SAMPLE CORRECTING HX SAMPLE *
959								* $HX = HX(MFAC) + HY(MEAS)*SIN(ERKOR)$ *
960								* LOCATE SIN(ERROR) IN SINE *
961								* USES "SMPLY" TO FORM PRODUCT & "SADD" TO SUM *
962								* EXIT WITH MODIFIED HX IN DATM,DATM+1 *
963								*****
964	0E48	0E48	75	01				ORTH CPSL C
965	0E8A		0C	04	5C			LODA,R0 DATM+2 HY MSB
966	0E8D		0D	04	5D			LODA,R1 DATM+3 HY LSB
967	0EC0		0E	07	1E			LODA,R2 SINE SIN(ERROR)
968	0EC3		C3					STRZ R3
969	0EC4		44	7F				ANDI,R0 H17F1 STRIP SIGN
970	0EC6		47	80				ANDI,R3 H1801 SAVE SIGN
971	0EC8		CC	04	7B			STRA,R0 OPR2
972	0ECH		CD	04	7C			STRA,R1 OPR2+1
973	0ECE		CE	04	7A			STRA,R2 OPR1+1
974	0ED1		CF	04	7E			STRA,R3 SOES SIGN OF HY*SIN(FRR)
975	0ED4		3F	08	66			BSTA,UN SMPLY FORM PROD.
976	0ED7		0C	04	5A			LODA,R0 DATM HX MSB
977	0EDA		0D	04	5B			LODA,R1 DATM+1 HX LSB
978	0EDD		C2					STRZ R2
979	0EDF		44	7F				ANDI,R0 H17F1
980	0EE0		46	80				ANDI,R2 H1801
981	0EE2		CC	04	65			STRA,R0 TFMP
982	0EE5		CD	04	66			STRA,R1 TFMP+1
983	0EE8		CE	04	7D			STRA,R2 STEM
984	0EEB		75	02				CPSL COM
985	0EED		3F	09	48			BSTA,UN SADD FORM SUM
986	0EEF		0C	04	77			LODA,R0 RSLT
987	0EF3		0D	04	7B			LODA,R1 RSLT+1
988	0EF6		CC	04	5A			STRA,R0 DATM MODIFIED HX
989	0EF9		CD	04	5B			STRA,R1 DATM+1
990	0EFC		77	02				PPSL COM
991	0EFE		17					RSLT,UN
992								*****
993								END

TOTAL ASSEMBLY ERRORS = 0

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## APPENDIX C

The transcendental functions used throughout the heading computation algorithm were implemented using a table look up procedure. To generate the respective look up tables in computer memory data was first generated using algol programs. This technique expedited modifications to tabular data and provided output data in a convenient (hexadecimal) format.

Programs that calculated  $\cos(\theta)$  and  $\cos^2(\theta)$  to eight bit and sixteen bit resolution respectively are included.

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PAGE 001

```

001 00000 HPAI ,L,"RP"
002 00000 BFGT:
003 00001 A*****
004 00001 A A PROGRAM TO GENERATE A COS-COS TABLE FOR A *
005 00001 A MICROMINOCSSOR BASED SYSTEM
006 00001 A ADJUST SS+ ANGLE , COS(ANGLE), AND DATA IN
007 00001 A BOTH BINARY AND HEX FORMAT IS TABULATED
008 00001 A*****
009 00001 INTEGRAL A,B,C,D,E,F
010 00010 REAL THETA,STHETA,AN,HSTH
011 00020 REAL ARRAY AN(0:256,0:15)
012 21070 INTEGRAL ARRAY VAL(0:15)="0","1","2","3","4","5",
013 21102 "6","7","8","9","A","B",
014 21107 "C","D","E","F"
015 21114 INTEGRAL ARRAY MUL(0:3)=8,4,2,1
016 21124 INTEGRAL ARRAY HEX(0:256,0:3)
017 23136 WRITE (A,*(//,9X,"AN,5X,"THETA",4X,"COS-COS",5X,
018 23175 "R[160Y",14X,"HEX",//))
019 23226 "R[160Y",14X,"HEX",//))
020 23242 WRITE (A,*(5X,"(A000)",3X,"(000)",18X," DATA ",//))
021 23276 FOR A:= 45 TO 90 DO
022 23304 BEGIN
023 23304 AN:=A
024 23310 THETA:= ARCP(1/180)
025 23316 STHETA:=(COS(THETA))^2
026 23330 H:=0
027 23332 STH:=STHETA
028 23336 H(1)=H+1/DO
029 23341 BEGIN
030 23342 STH:=2*H*STH
031 23350 IF STH>1 THEN
032 23355 BEGIN
033 23355 AN(A,H)=1
034 23367 STH=STH-1
035 23375 ENIF ELSE AN(A,H)=0
036 23435 H:=H+1
037 23440 ENIF
038 23441 H:=0
039 23443 FOR C=0 TO 3 DO
040 23451 BEGIN
041 23451 I:=4-C
042 23455 F:=0
043 23457 FOR F:=0 TO 3 DO
044 23465 BEGIN
045 23465 F:=F+MUL(F)*AN(A,I+I)
046 23520 HEX(A,C):= VAL(F)
047 23533 ENIF
048 23537 ENIF
049 23543 THETA:= THETA*180/PI
050 23553 WRITE (B,*(5X,15,2(3X,F2,6),3X,10I1,5X,4A2),A,
051 23604 THETA,STHETA, FOR H:= 0 TO 15 DO
052 23620 AN(A,H), FOR I:= 0 TO 3 DO HEX(A,I))
053 23653 ENIF
054 23657 ENDO

```

PROGRAM= 02766J FROMPS=000

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A	THETA	COS*COS	BINARY	HEX
(ADDR.)	(DEG)		DATA	
45	45.00000	.500000	0111111111111111	7 F F F
46	46.00000	.482550	0111011100010000	7 8 8 8
47	47.00000	.465122	0111011100010010	7 7 1 2
48	48.00000	.447736	0111001010011110	7 2 9 E
49	48.99999	.430413	0110111000101111	6 E 2 F
50	50.00000	.413176	0110100111000101	6 9 C 5
51	51.00000	.396044	0110010101100011	6 5 6 3
52	52.00001	.379039	0110000100001000	6 1 0 8
53	53.00000	.362181	0101110010110111	5 C 8 7
54	54.00000	.345491	0101100001110010	5 8 7 2
55	55.00000	.328990	0101010000111000	5 4 3 8
56	56.00000	.312697	0101000000001100	5 0 0 C
57	57.00000	.296672	0100101111110000	4 B F 0
58	58.00000	.280814	0100011111100011	4 7 E 3
59	58.99999	.265264	0100001111101000	4 3 E 8
60	60.00001	.250000	0011111111111111	3 F F F
61	61.00000	.235040	0011110000101011	3 C 2 8
62	62.00000	.220403	0011100001101100	3 8 6 C
63	63.00000	.206107	0011010011000011	3 4 C 3
64	64.00000	.192169	0011000100110010	3 1 3 2
65	65.00000	.178606	0010110110111001	2 0 8 9
66	66.00000	.165435	0010101001011001	2 A 5 9
67	66.99998	.152671	0010011100010101	2 7 1 5
68	68.00000	.140370	0010001111101100	2 3 E C
69	69.00000	.128427	0010000011100000	2 0 E 0
70	70.00000	.116978	0001110111110010	1 0 F 2
71	71.00000	.105995	0001101100100010	1 8 2 2
72	72.00000	.095491	0001100001110010	1 8 7 2
73	73.00000	.085481	0001010111100010	1 5 E 2
74	73.99998	.075476	0001001101110011	1 3 7 3
75	75.00000	.066987	0001000100100110	1 1 2 6
76	76.00000	.058526	0000111011111011	0 E F 8
77	77.00002	.050603	0000110011110100	0 C F 4
78	78.00000	.043227	0000101100010000	0 8 1 0
79	79.00000	.036408	0000100101010010	0 9 5 2
80	80.00000	.030154	0000011110111000	0 7 8 8
81	81.00000	.024472	0000011001000011	0 6 4 3
82	82.00000	.019349	0000010011110101	0 4 F 5
83	83.00000	.014852	0000001111001101	0 3 C 0
84	84.00002	.010926	0000001011001100	0 2 C C
85	86.99998	.007596	0000000111110001	0 1 F 1
86	86.00000	.004866	0000000100111110	0 1 3 F
87	87.00000	.002739	0000000010110011	0 0 8 3
88	88.00000	.001218	0000000001001111	0 0 4 F
89	89.00000	.000305	0000000000100111	0 0 1 J
90	90.00000	.000000	0000000000000000	0 0 0 0

PAGE 001

```

001 00000 HPA1,1,"RP"
002 00000 HPGIN
003 00001 A*****
004 00001 A A PROGRAM TO GENERATE A COS TABLE FOR A
005 00001 A MICROPROCESSOR BASED SYSTEM
006 00001 A ADDRESS, ANGLE, COS(ANGLE), AND DATA IN
007 00001 A BOTH BINARY AND HEX FORMAT IS TABULATED
008 00001 A*****
009 00001 INTEGER A,B,C,D,E,F;
010 00010 RFA1 THETA,STHETA,ASTH1
011 00020 RFA1 ARRAY AN(0:256,0:16);
012 21070 INTEGER ARRAY VAL(0:15);="0","1","2","3","4","5",
013 21102 "6","7","8","9","A","B",
014 21107 "C","D","E","F";
015 21114 INTEGER ARRAY MULT(0:15);="8","4","2","1";
016 21124 INTEGER ARRAY HEX(0:256,0:15);
017 23136 WRITE (A,*(I,1));
018 23175 WRITE (6,*(//,9X,"A",5X,"THETA",4X,"COS(THETA)",2X,"BINARY",6X,"HEX",
019 23240 /));
020 23243 WRITE (6,*(9X,"(ADDR.)",3X,"(DEG)",18X," DATA ",//));
021 23277 FOR A = 45 TO 90 DO
022 23305 BEGIN
023 23305 AK:=A;
024 23311 THETA:= AK*PI/180;
025 23317 STHETA:= COS(THETA);
026 23323 A:=0;
027 23325 ASTH:=STHETA;
028 23331 WHILE LF H<0 DO
029 23334 BEGIN
030 23335 ASTH:= 2*ASTH;
031 23343 IF ASTH>1 THEN
032 23350 BEGIN
033 23350 AN(A,0):=1;
034 23362 ASTH:=ASTH-1;
035 23370 END ELSE AN(A,B):=0;
036 23403 A:=B+1;
037 23406 END;
038 23407 B:=0;
039 23411 FOR C:=0 TO 3 DO
040 23417 BEGIN
041 23417 B:=4*C;
042 23450 F:=0;
043 23452 FOR E:=0 TO 7 DO
044 23460 BEGIN
045 23460 F:=F*MULT(F)*AN(A,B+E);
046 23513 HEX(A,C):= VAL[F];
047 23526 END;
048 23532 END;
049 23536 THETA:= THETA*180/PI;
050 23546 WRITE (6,*(15X,15,2(3X,FR.6),3X,8F1.0,5X,PA2),A,THETA,STHETA,
051 23605 FOR H:=0 TO 7 DO AN(A,B),FOR D:=0 TO 1 DO HEX(A,D));
052 23646 END;
053 23652 END

```

PROGRAM= 027656 FROWS=000

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A (ADDR.)	THETA (DEG)	COS(THETA)	BINARY DATA	HEX
45	45.00000	.707107	1.11.1.1	8 5
46	46.00000	.694659	1.11...1	8 1
47	47.00000	.681998	1.1.111.	A E
48	48.00000	.669131	1.1.1.11	A 8
49	48.99999	.656059	1.1..111	A 7
50	50.00000	.642788	1.1..1..	A 4
51	51.00000	.629320	1.1....1	A 1
52	52.00001	.615662	1..111.1	9 0
53	53.00000	.601815	1..11.1.	9 A
54	54.00000	.587785	1..1.11.	9 6
55	55.00000	.573576	1..1..1.	9 2
56	56.00000	.559193	1...1111	8 F
57	57.00000	.544639	1...1.11	8 8
58	58.00000	.529919	1....111	8 7
59	58.99999	.515039	1.....11	8 3
60	60.00001	.500000	.1111111	7 F
61	61.00000	.484810	.11111..	7 C
62	62.00000	.469471	.1111...	7 8
63	63.00000	.453991	.111.1..	7 4
64	64.00000	.438371	.111....	7 0
65	65.00000	.422619	.11.11..	6 C
66	66.00000	.406737	.11.1...	6 8
67	66.99998	.390731	.11..1..	6 4
68	68.00000	.374607	.1.11111	5 F
69	69.00000	.358345	.1.11.11	5 8
70	70.00000	.342020	.1.1.111	5 7
71	71.00000	.325568	.1.1..11	5 3
72	72.00000	.309017	.1..1111	4 F
73	73.00000	.292372	.1..1.1.	4 A
74	73.99998	.275637	.1...11.	4 6
75	75.00000	.258319	.1....1.	4 2
76	76.00000	.241922	..1111.1	3 0
77	77.00002	.224951	..111.1.	3 9
78	78.00000	.207912	..11.1.1	3 5
79	79.00000	.190809	..11....	3 0
80	80.00000	.173648	..1.11..	2 C
81	81.00000	.156435	..1.1...	2 8
82	82.00000	.139173	..1...11	2 3
83	83.00000	.121869	...11111	1 F
84	84.00002	.104528	...11.1.	1 A
85	84.99998	.087156	...1.11.	1 6
86	86.00000	.069757	...1...1	1 1
87	87.00000	.052336	....11.1	0 0
88	88.00000	.034900	....1...	0 8
89	89.00000	.017452	.....1..	0 4
90	90.00000	-.000000	.....	0 0

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### Section III

Earth Electric Field Research -  
& Field Model of Cumulonimbus Cloud

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## I. INTRODUCTION

The fact that the Earth has a surrounding electric field has been known for centuries, but the study of the effect of clouds on this electric field is a fairly 'new area of study. Ideally the earth's electric field is a perfect vertical, however since clouds are not electrically neutral, their presence distorts the earth's field; nearly neutral clouds causing slight perturbations while heavily charged clouds actually reverse the direction of the resultant field. The most highly charged clouds belong to the cumulonimbus family.

The report evaluates various electrical models of cumulonimbus clouds. The resultant field of each model is compared with actual readings of the earth's electric field in the presence of a cumulonimbus cloud at various heights and distances from the cloud. Using the actual field readings, this report will develop a new electrical model of a cumulonimbus cloud.

This paper also reviews the electrical properties of the atmosphere, particularly of clouds and suggests some possible uses of the knowledge of the field around a cumulonimbus cloud.

## II. ATMOSPHERIC ELECTRICITY

Atmospheric electricity is defined by Dolezalek [1] as "the science of everything electric in the atmosphere between the solid and liquid surface of the earth to the lower regions of the ionosphere, including the boundary layers of both."

Like every science, atmospheric electricity has its own sign conventions. Distance is measured positively upward. Current flow to the earth is considered positive. Electric potential is measured relative to earth. The terms "field" and "potential gradient" are in general used to denote the same quantity, i.e. the partial derivative of voltage with respect to height. The units used are the MKSA system.

All lines of force commence on a positive charge and end on a negative charge. The density of lines of force across any area gives a measure of the field. If the line of force terminates on the earth's surface, it must enter vertically. Any change in space charge will immediately effect the lines of force and thus, the electric field.

As space charge appears, a potential gradient is produced immediately at the ground. The air-earth current will change as will the lines of force. To determine the potential gradient change produced due to the appearance of the space charge, one must know the relaxation time,  $\epsilon/\lambda$ , of the air where  $\epsilon$  is the permittivity of the air and  $\lambda$  is the

conductivity. Near the earth's surface the relaxation time generally ranges between 5 to 40 minutes depending to a great extent on the pollution level. The higher the pollution level, the lower the conductivity and, therefore, the higher the relaxation time. At an altitude of 18km the relaxation time is about 4 seconds while at 70km it is on the order of  $10^{-8}$  seconds. At the earth's surface it is  $10^{-6}$  seconds or less. The potential gradient at some time after a change in space charge can be given by the equation [2]

$$E = E_1 e^{-t/\tau} + E_2 (1 - e^{-t/\tau}) \quad (1)$$

where  $E_1$  is the potential gradient immediately after the change,  $E_2$  is the potential gradient when the conditions are again quasistatic,  $\tau$  is the relaxation time and  $E$  is the potential gradient in the intervening time.

Ions are relatively rare when compared to the total number of molecules in the atmosphere. A cubic meter of clean land air will contain approximately 800 million ions out of  $10^{25}$  molecules. The conductivity of the atmosphere may be written as [1]

$$\lambda = \sum n_i^+ e_i^+ k_i^+ + \sum n_j^- e_j^- k_j^- \quad (2)$$

where  $n_i^+$  is the number of positive ions with a charge of  $e_i^+$  and a mobility of  $k_i^+$ ,  $n_j^-$  is the number of negative ions with a charge of  $e_j^-$  and a mobility of  $k_j^-$ , and  $\lambda$  is the conductivity. The mobility

is computed from the potential gradient  $E$  and the velocity of the particle  $v$  by

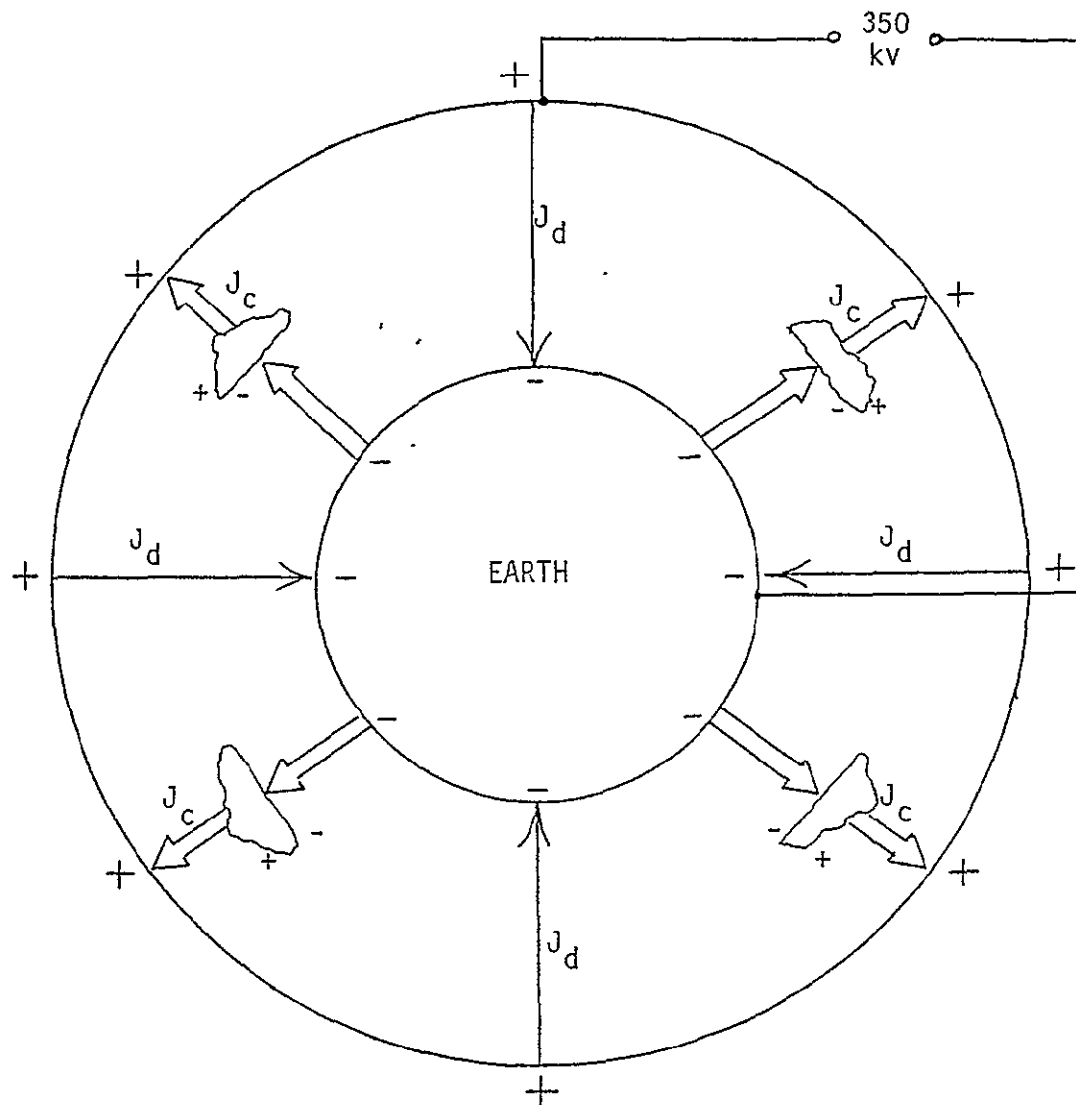
$$k = v / E \quad (3)$$

or the velocity acquired in a potential gradient of 1 volt/meter. The mobility is highly dependent on physical properties of the ion such as size and shape.

Since the air-earth current is continually bringing positive charges to the earth there must be some restoring component which will complete the circuit carrying the positive charges upward again. The most popular theory on the generation and continuation of the electrical characteristics of the atmosphere is the condenser theory. [1,3] In this theory the ionosphere is positively charged with respect to the earth. The air-earth current tends to continually discharge this global condenser. However, this process is compensated by thunderstorms which act as the generator restoring positive charge to the ionosphere. Figure 1 shows the schematic of this global circuit. The lower portion of a thunderstorm is negatively charged while the upper portion is positive. The world wide occurrence of thunderstorms also corresponds to the diurnal variation of the earth's electric field.

The study of atmospheric electricity is divided into two quite different divisions, fair weather and disturbed weather. Disturbed weather is defined by Dolezak as "when we have hydrometers in the atmosphere - fog, rain, snow - or when we have high winds, in particular in





$J_d$  Discharging Currents

$J_c$  Charging Currents

Figure 1

(from IEEE Spectrum [3])

connection with blown-up dust or snow, or when we have much of the sky covered by substantial clouds." [1] The absence of these phenomena is considered fair weather. Fair weather conditions are predictable and one can assume a quasistatic state to determine its properties, while disturbed weather conditions can alter rapidly and a quasistatic state cannot be applied.

Fair weather conditions have several characteristic quantities. Voltage increases with height. The potential gradient is positive and essentially vertical. It is constant for the first few meters and shows a progressive decrease with height at approximately 100 meters. The conductivities are also constant for the first few meters, then show a marked increase with height at approximately 100 meters due to an increase with altitude of ionization by cosmic rays and a decrease in pollution. The current density is the same at all levels and is equal to the voltage of the ionosphere with respect to earth divided by the resistance of a  $1\text{m}^2$  column of air from the earth to the ionosphere. Tables 1 and 2 list fair weather parameters as compiled by Dolezelek. [1]

The disturbed weather phenomena is much harder to typify. The potential gradient can have horizontal variations of large magnitude. There are added currents carried by precipitation and lightning. Constant changes in conductivity occur due to fresh charges. It is in this still unsolved phenomena that much of the current work is being performed.

Part of Atmos- phere for which the Values are Calculated	Currents, I, in A; and current densities, i, in A/m <sup>2</sup>	Potential Differences, U, in V; field strength E in V/m	Resistances, R, in $\Omega$ ; Columnar res., R <sub>C</sub> , in $\Omega$ m <sup>2</sup> Resistivities, $\rho$ , in $\Omega$ m
Volume element at about sea level, one cubic meter	$i=3 \times 10^{-12}$	$E_0=1.2 \times 10^2$	$\rho_0=4 \times 10^{13}$
Lower column of 1 m <sup>2</sup> cross section - sea- level to 2 km	same as above	at upper end: $U_1=1.8 \times 10^5$	$R_{C1}=6 \times 10^{16}$
Volume element at about 2 km height, 1 m <sup>3</sup>	same as above	$E_2=6.6 \times 10^1$	$\rho_2=2.2 \times 10^{13}$
Center column of 1 m <sup>2</sup> cross section - 2 to 12 km	same as above	at upper end: $U_m=3.15 \times 10^5$	$R_{cm}=4.5 \times 10^{16}$
Volume element at about 12 km height, 1 m <sup>3</sup>	same as above	$E_{12}=4.2 \times 10^0$	$\rho_{12}=1.3 \times 10^{12}$
Upper column of 1 m <sup>2</sup> cross section - 12 to 65 km	same as above	at upper end: $U_u=3.5 \times 10^5$	$R_{cu}=1.5 \times 10^{16}$
Whole column of 1 m <sup>2</sup> cross section - 0 to 65 km	same as above	at upper end: $U=3.5 \times 10^5$	$R_C=1.2 \times 10^{17}$
Total spher- ical capacitor area: 5x10 <sup>14</sup> m <sup>2</sup>	$I=1.5 \times 10^3$	$U=3.5 \times 10^5$	$R=2.4 \times 10^2$

Table 1

Part of Atmosphere for which the Values are Calculated	Conductances, G, in $\Omega^{-1}$ ; Col. conductances $G_c$ , in $\Omega^{-1}m^{-2}$ ; total cond. $\Lambda$ , in $\Omega^{-1}m^{-1}$	Capacitances, C, in F; Col. capacitances $C_c$ , in $Fm^{-2}$ ; Capacitivities $\epsilon$ , in $Fm^{-1}$	Relaxation Times $\tau$ , in seconds
Volume element at about sea level, one cubic meter	$\Lambda_0 = 2.5 \times 10^{-14}$	$\epsilon_0 = 8.9 \times 10^{-12}$	$\tau_0 = 3.6 \times 10^2$
Lower column of $1 m^2$ cross section - sea level to 2 km	$G_{c1} = 1.7 \times 10^{-17}$	$C_{c1} = 4.4 \times 10^{-15}$	$\tau_{c1} = 2.6 \times 10^2$
Volume element at about 2 km height, $1 m^3$	$\Lambda_2 = 4.5 \times 10^{-14}$	$\epsilon_2 = 8.9 \times 10^{-12}$	$\tau_2 = 2 \times 10^2$
Center column of $1 m^2$ cross section - 2 to 12 km	$G_{cm} = 5 \times 10^{-17}$	$C_{cm} = 8.8 \times 10^{-16}$	$\tau_{cm} = 1.8 \times 10^1$
Volume element at about 12 km height, $1 m^3$	$\Lambda_{12} = 4.0 \times 10^{-13}$	$\epsilon_{12} = 8.9 \times 10^{-12}$	$\tau_{12} = 1.2 \times 10^1$
Upper column of $1 m^2$ cross section - 12 to 65 km	$G_{cu} = 2.5 \times 10^{-17}$	$C_{cu} = 1.67 \times 10^{-16}$	$\tau_{cu} = 6.7 \times 10^0$
Whole column of $1 m^2$ cross section - 0 to 65 km	$G_c = 8.3 \times 10^{-18}$	$C_c = 1.36 \times 10^{-16}$	$\tau_c = 1.64 \times 10^1$
Total spherical capacitor area: $5 \times 10^{14} m^2$	$G = 4.2 \times 10^{-3}$	$C = 6.8 \times 10^{-2}$	$\tau = 1.64 \times 10^1$

Table 2

### III. METEOROLOGY

The nomenclature for the various sections of the atmosphere are not agreed upon by all who study the atmosphere, however the divisions and their names as used by Dobson [4] are shown in Figure 2.

The lowest cloud in the troposphere is known as fog. This occurs when the ground is within a few degrees of the dew point. Fog can be produced three ways. Radiation fog is created on clear nights if the earth cools greatly since without a cloud cover the heat radiates into the upper atmosphere. A slight breeze will then bring the earth to the dew point. Advection fog is caused by warm moist air blowing over a colder surface and becoming chilled to its dew point. Frontal fog may be produced when cold air mixes with warm moist air. Other cloud types are summarized in Table 3.

The cloud names come from four basic words: cirrus, cumulus, stratus and nimbus. Cirrus clouds are composed of ice crystals and are delicate and curly in appearance. Sun shines through them without a shadow. Cumulus clouds are lumpy or billowing forms. The sheetlike cloud layers are stratus clouds. Nimbus clouds are stratus or cumulus that develop a "head" or thickness. "Alto" is also used as a prefix to indicate clouds at intermediate heights.

When two air masses of differing temperatures, pressures and relative humidities mix, a front is formed. The colder air mass sits

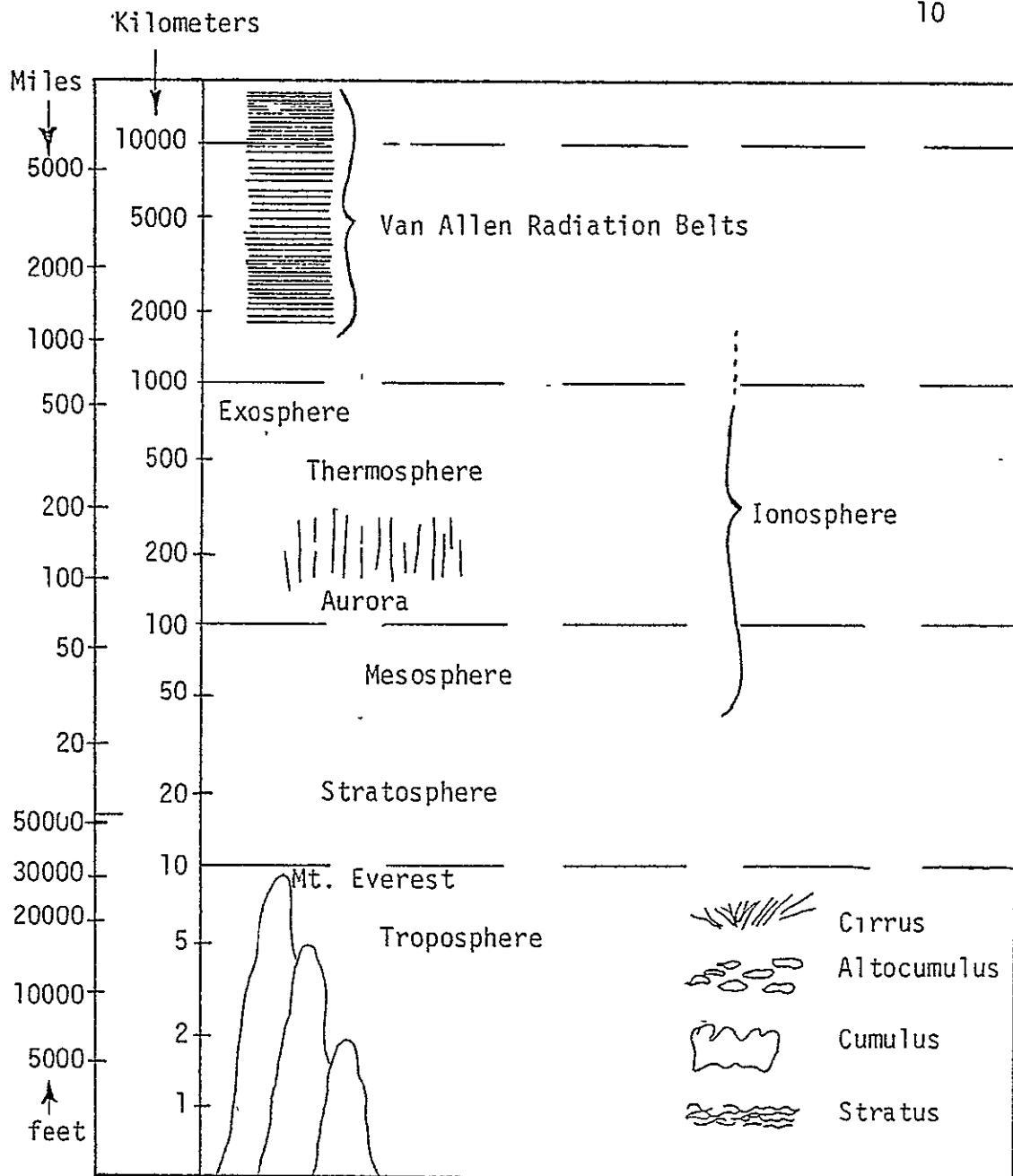


Figure 2

<u>CLOUD NAME</u>	<u>AVERAGE HEIGHT AND RANGE IN HEIGHT</u>
<u>High Clouds</u>	
Cirrus	30,000 ft. (10,000 to 50,000 ft.)
Cirro-cumulus	20,000 ft. ( 8,000 to 36,000 ft.)
Cirro-stratus	35,000 ft. (15,000 to 45,000 ft.)
<u>Middle Clouds</u>	
Alto-cumulus	12,000 ft. ( 3,000 to 27,000 ft.)
Alto-stratus	15,000 ft. ( 5,000 to 35,000 ft.)
<u>Low Clouds</u>	
Strato-cumulus	5,000 ft. ( 1,000 to 15,000 ft.)
Stratus	2,000 ft. ( 50 to 6,000 ft.)
Nimbo-stratus	2,500 ft. ( 200 to 18,000 ft.)
<u>Clouds with Vertical Development</u>	
Cumulus	2,500 ft. ( 1,000 to 10,000 ft.) Tops may extend to 20,000 ft.
Cumulo-nimbus	2,500 ft. ( 500 to 10,000 ft.) Tops may extend to over 35,000 ft.

Table 3

(from Aerology for Pilots [5])

wedgelike under the warmer air mass. The air that is displacing another air mass gives its name to the front; thus, if cold air is displacing warm air it is known as a cold front. Likewise, in a warm front warm air displaces the colder air. An occluded front exists when two colder air masses trap a third air mass between them forcing the third air mass aloft until it dissipates. Figure 3 shows cross sections of the warm and cold types of fronts and the clouds associated with them.



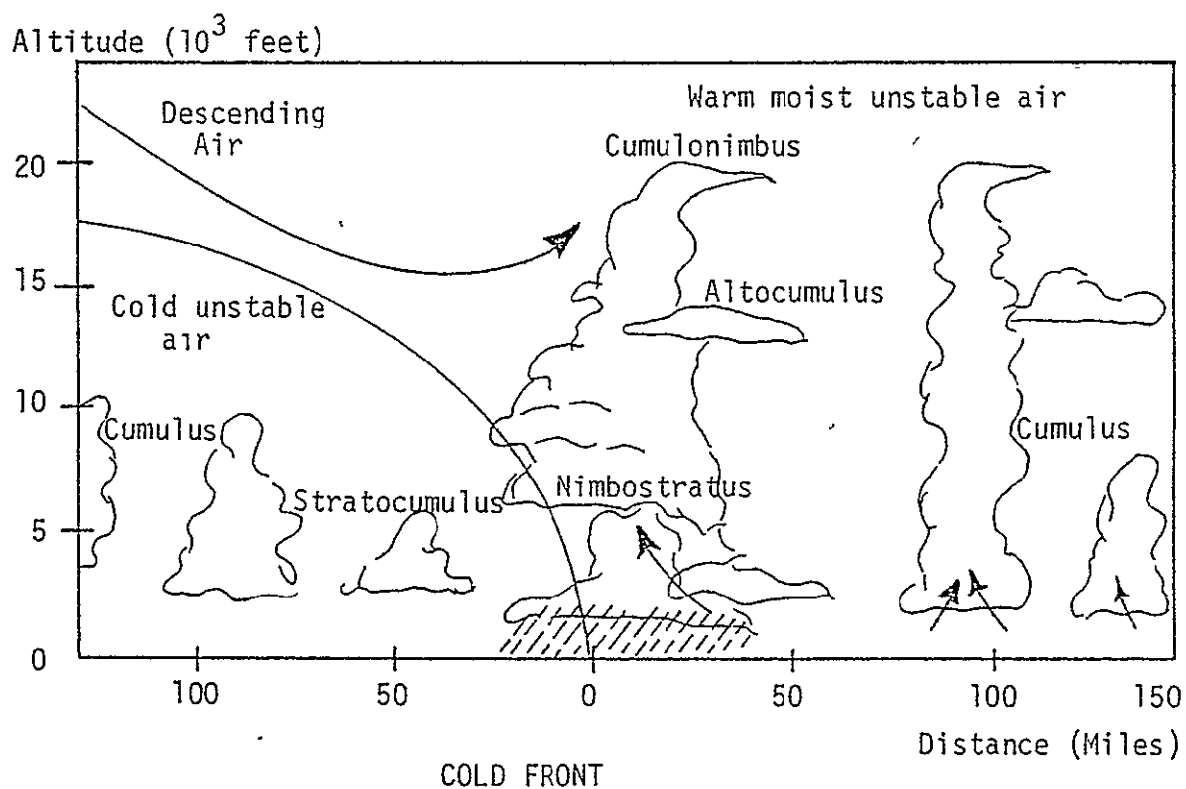
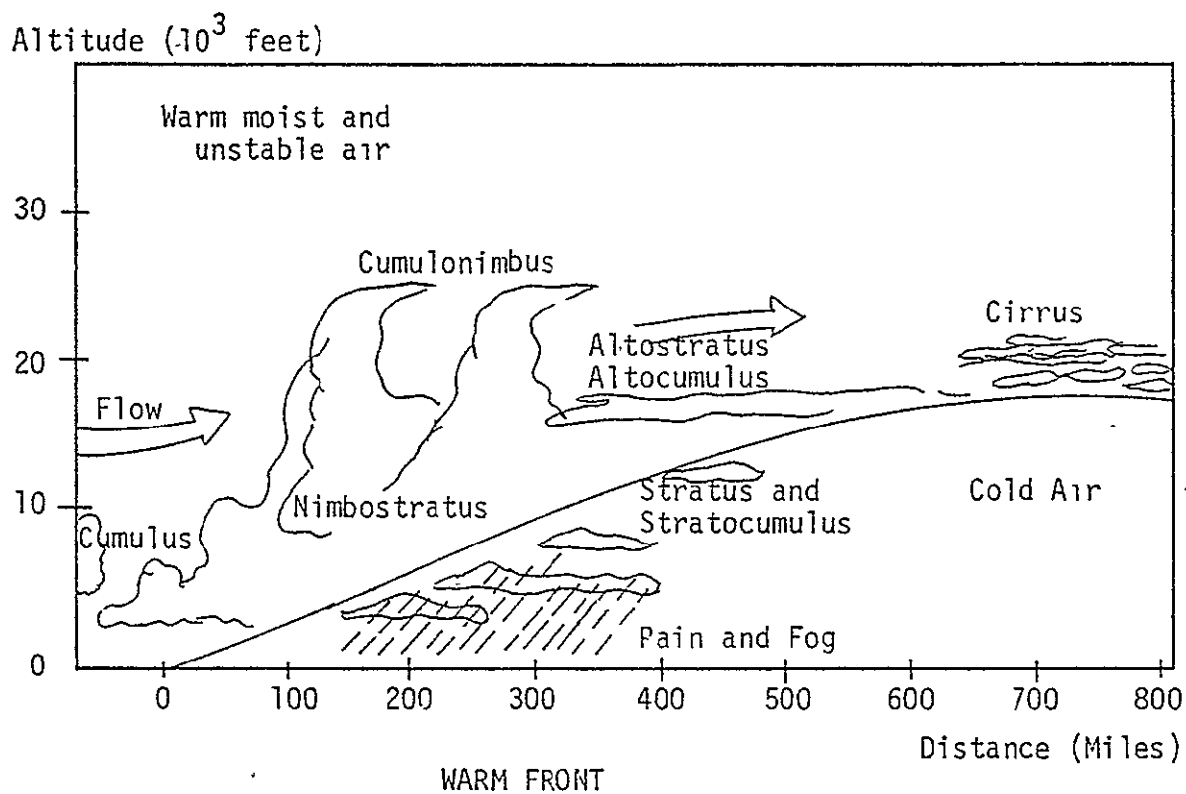


Figure 3

(from Introduction to Aviation [6])

#### IV. EXISTING MODELS

The cumulonimbus cloud differs from other types of clouds in its rate of accumulation of charge. A collection of charges, such as a cloud, in a conductive medium will attract an equal but oppositely charged screening layer and, therefore, cause no appreciable change in the electric field. However, since the rate of increase of charge in a cumulonimbus cloud within the time constant of the surrounding atmosphere approximately doubles the amount of charge [7], an effective screening layer can not be generated and, therefore, the cloud effects the neighboring electric field. Models of cumulonimbus clouds can eliminate the screening layer and simply use the net increase in charge as the only charge present.

Experimentors have been attempting for years to gather data on the electric field in the vicinity of a thundercloud, i.e. a cumulonimbus cloud, in order to generate a valid electrical model for the cloud. The measurements were originally made at ground level but more recently include airborne measurements in, around and above thunderclouds.

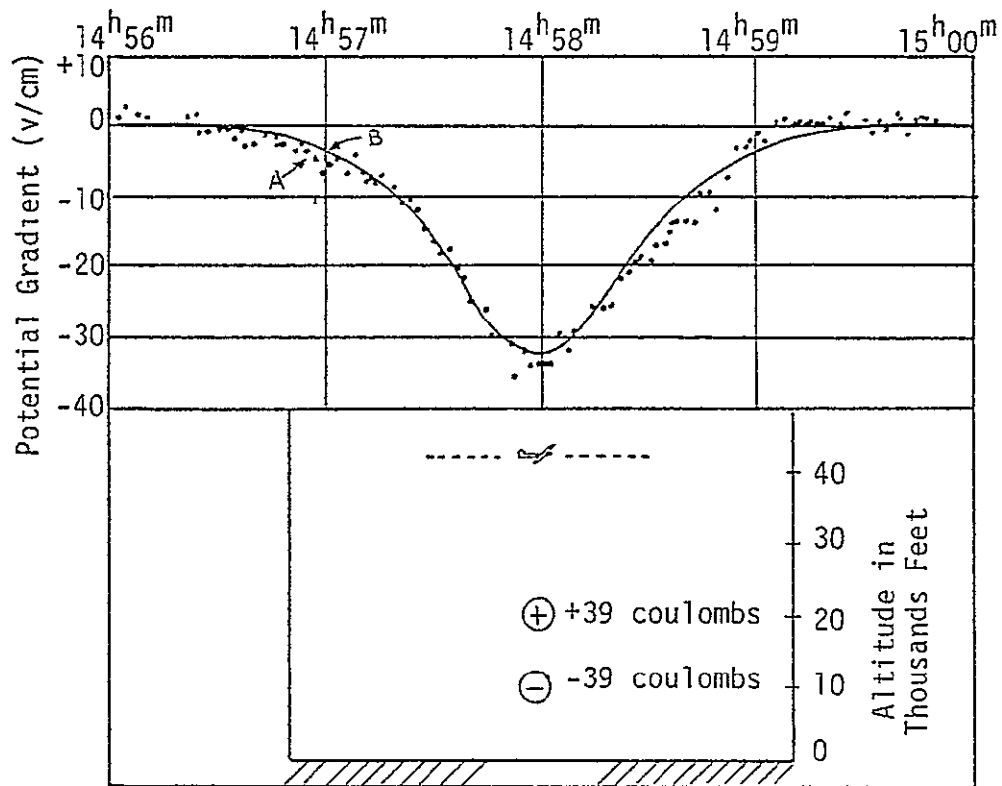
In 1948 O.H. Gish and G.R. Wait [8], in an effort to show that thunderclouds could supply the negative current flow necessary to maintain the general electrification of the earth, gathered measurements of the electric field strength above thunderstorms. Their findings were as follows: Of 87 traverses across thunderstorms, in 22 data was

incomplete or unsatisfactory, 31 profiles corresponded well to a bipolar model while the remaining 34 were of a more complicated nature where a bipolar model was only a good first approximation, additional dipoles would be required for an accurate model.

A model was proposed using the data gathered on a flight at an altitude of 43,000 feet on October 28, 1948. This data was typical of a storm when little lightning activity was present. The data and associated model are shown in Figure 4. A complete mapping of the electric field in an area 21km in horizontal distance from the cloud's charge concentrations and to a height of 16km is shown in Figure 5.

Israel [9] lists two other proposed electrical models for a thundercloud, one from Simpson and Robinson and the other from D.J. Malan. The model proposed by Simpson and Robinson is often referred to as the "Classical Model." It consists of a positive charge of 24 coulombs at a height of 6km, a charge of -20 coulombs at 3km and a charge of 4 coulombs at 1.5 km. The Malan model has a charge of 40 coulombs at approximately 10km, -40 coulombs at about 5km and 10 coulombs at about 2km. The electric field map for the Simpson and Robinson model is shown in Figure 6 and for the Malan model in Figure 7.

The program used to generate the data used in Figures 5, 6 and 7 and the data listings from the program are contained in Appendix A. The charge system for each of the three models used as inputs to the program are shown in Figure 10, utilizing the method of electrical images as described in the next section.

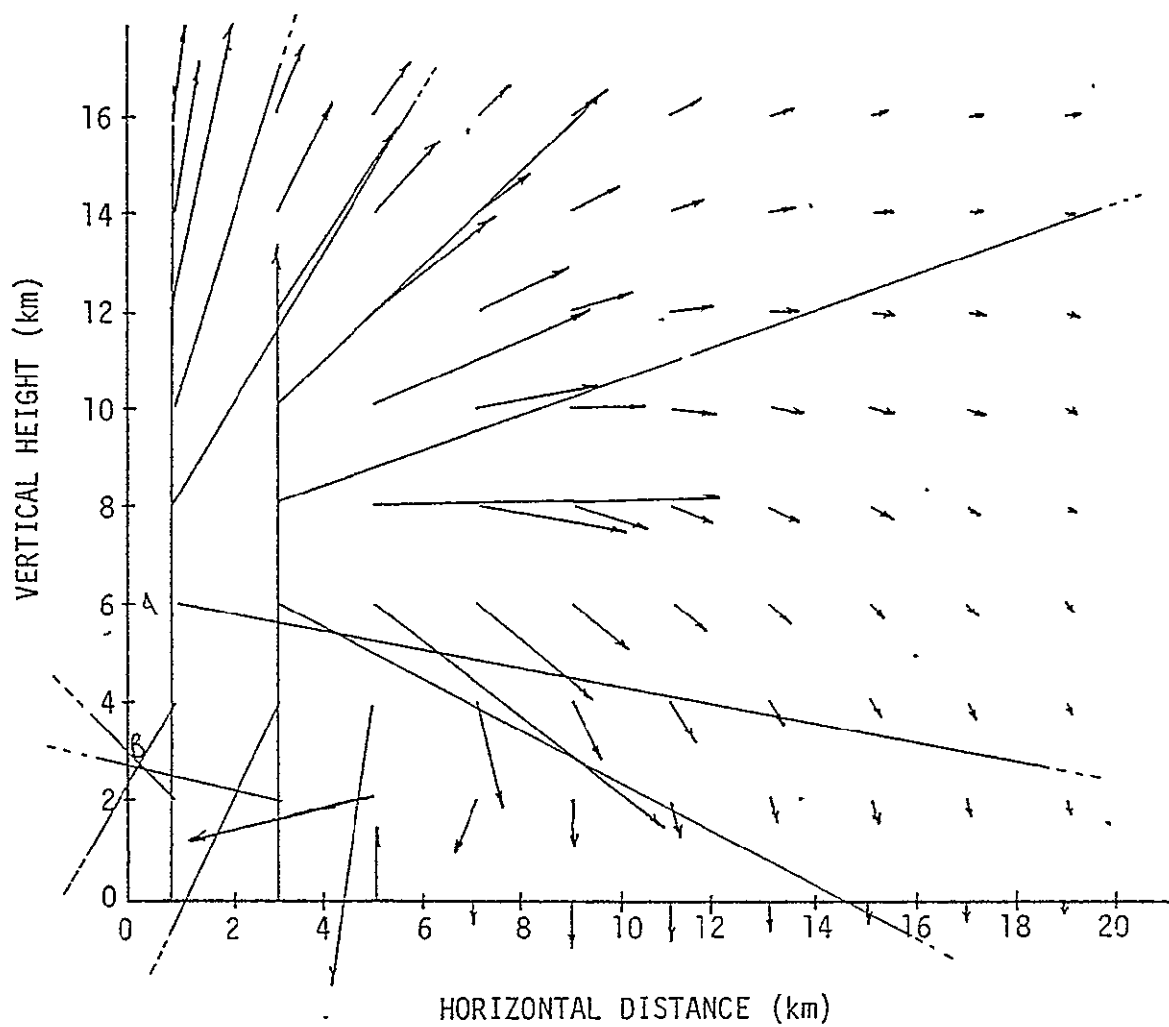


POTENTIAL GRADIENT (VERTICAL COMPONENT) OVER THUNDERSTORM,  
OCTOBER 28, 1948, STORM NO. 1, TRAVERSE NO. 8, (A) OBSERVED,  
(B) CALCULATED FOR MODEL SHOWN

Figure 4

(from Gish and Wait [8])

Scale: 1 In. = 4000 v/m

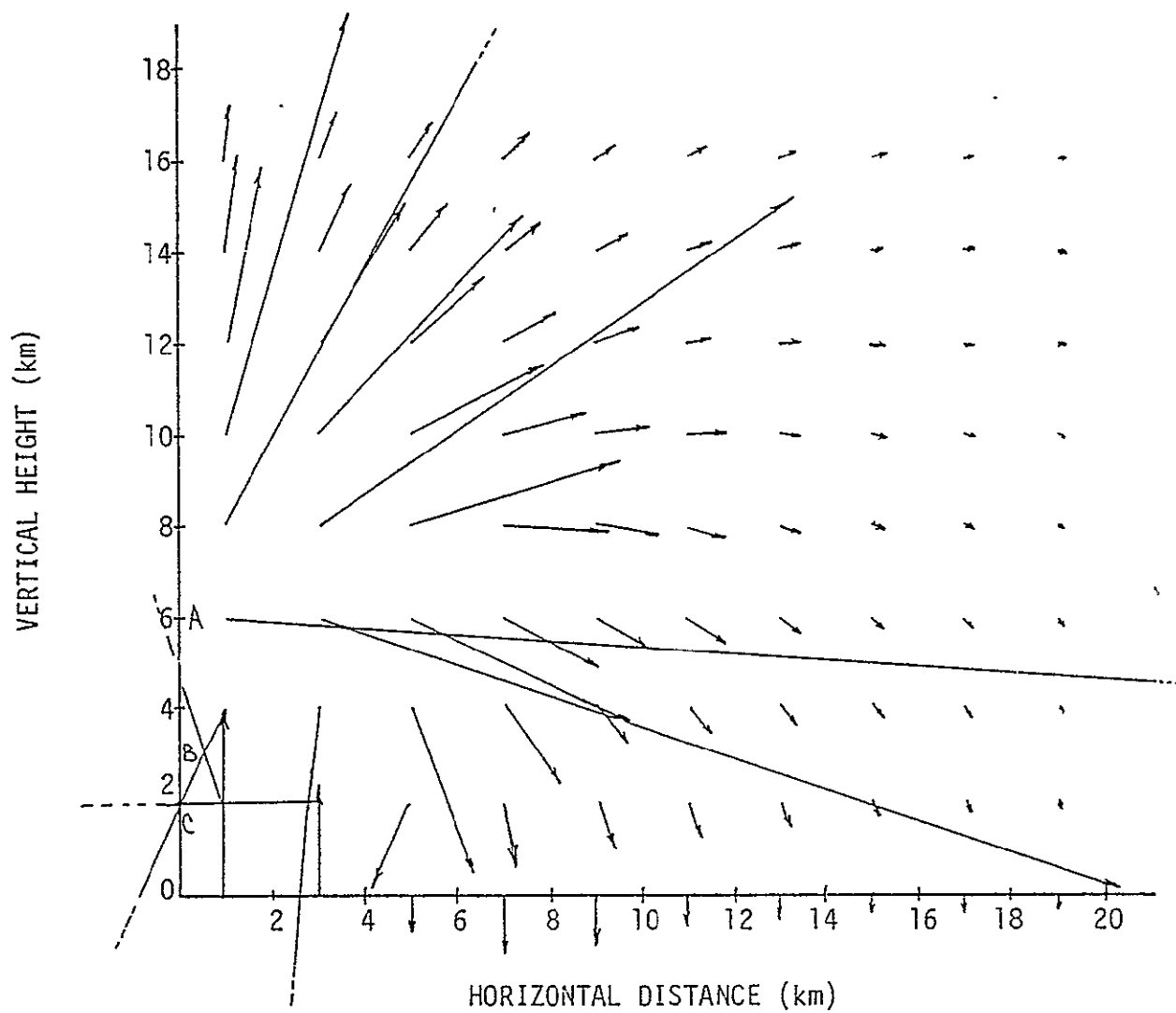


A = 39 coulombs  
B = -39 coulombs

ELECTRIC FIELD MAP FOR  
GISH AND WAIT MODEL

Figure 5

Scale: 1 in. = 4000 v/m



A = 24 coulombs  
B = -20 coulombs  
C = 4 coulombs

ELECTRIC FIELD MAP FOR  
SIMPSON AND ROBINSON MODEL

Figure 6

Scale: 1 in. = 4000 v/m

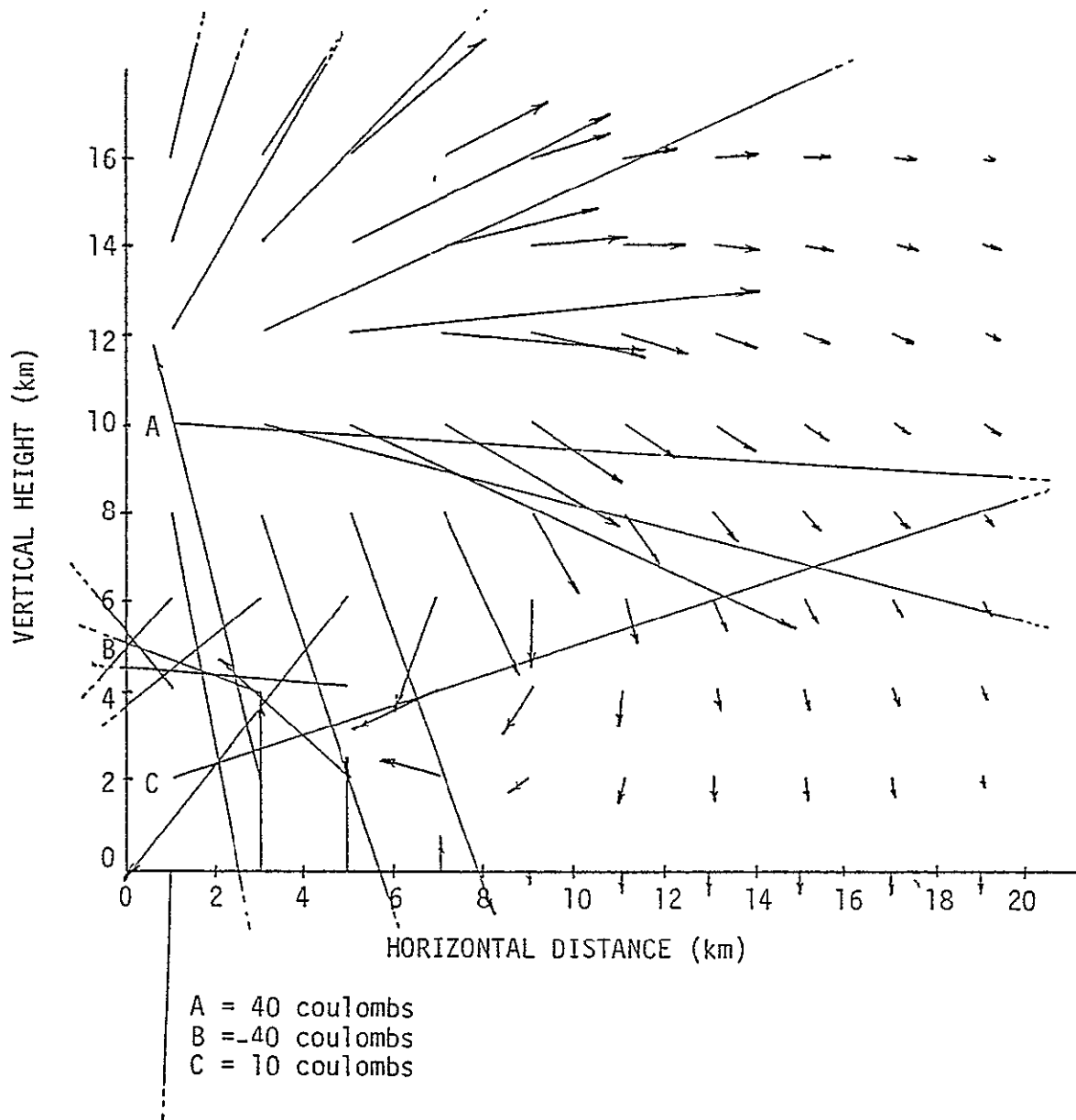
ELECTRIC FIELD MAP FOR  
MALAN MODEL

Figure 7

0-3.

## V. GENERAL NEAR-FIELD MODEL

The field at any point in space due to a single point charge  $Q$  can be computed from Gauss's law,

$$\oint_S \vec{D} \cdot d\vec{S} = \int_V \zeta \, dV \quad (4)$$

where  $\vec{D}$  is the electric flux density,  $V$  is the volume enclosed by the surface  $S$  and  $\zeta$  is the charge density within  $V$ . If one considers  $S$  to be a spherical surface with its center at  $Q$  ( $Q$  being the only charge within  $S$ ), the equation becomes

$$\vec{D} = (Q / 4\pi r^2) \vec{r} \quad (5)$$

Since  $\vec{D} = \epsilon \vec{E}$  where  $\epsilon$  is the permittivity of the substance then

$$\vec{E} = (Q / 4\pi r^2 \epsilon) \vec{r} \quad (6)$$

The direction of  $\vec{E}$  is radially outward if  $Q$  is positive and radially inward if  $Q$  is negative. See Figure 8 where  $Q$  is a positive charge. The magnitude of  $E$  is inversely proportional to the square of the distance to  $Q$ . If the permittivity is constant over all space concerned, the field created at a point  $P$  due to more than a single point charge may be computed by the vector addition

$$\vec{E} = \frac{1}{4\pi\epsilon} \left( \frac{Q_1}{r_{1P}^2} \vec{r}_{1P} + \frac{Q_2}{r_{2P}^2} \vec{r}_{2P} + \dots + \frac{Q_N}{r_{NP}^2} \vec{r}_{NP} \right) \quad (7)$$

where  $Q_J$  is the charge located at a point  $J$  and  $r_{JP}$  is the distance between  $Q_J$  and  $P$ . The permittivity used for this problem is that of free space, i.e.  $8.854 \times 10^{-12}$  farads per meter.



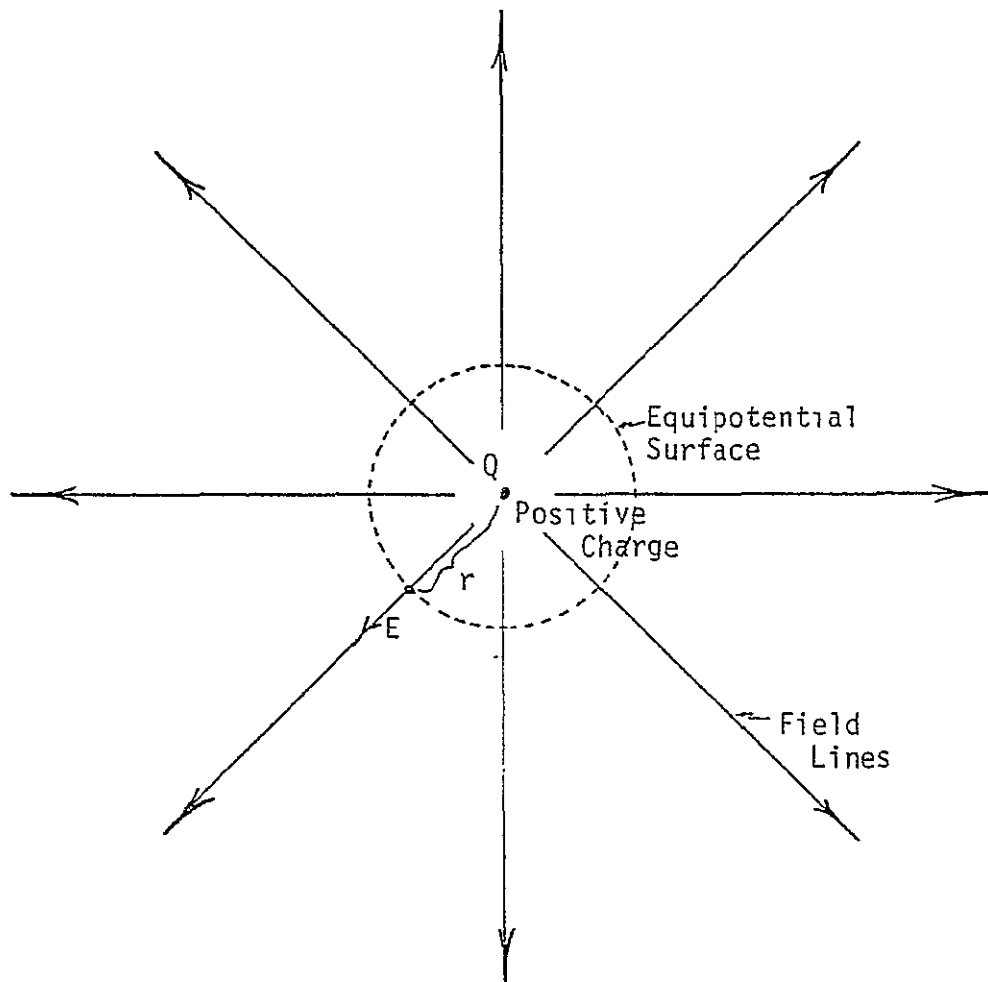


Figure 8

The earth, here considered flat over the area under consideration is an equipotential. To produce an equipotential surface one can use the method of electrical images, where an imaginary distribution of charges inside a conducting body of an electrostatic system is determined which would produce exactly the same field outside the body as that produced by the induced free charges over its surface. For the system of a conducting plane, one can look at a simplified problem of a single positive charge located above the plane. See Figure 9. As shown in the figure, an equal but oppositely charged point is placed an equal distance below the plane as the positive charge is above the plane. Now if the plane is removed, the field above the plane would remain unchanged. Therefore, the field produced by all the charges induced on the conducting plane may be reduced to the field produced by the single point,  $-Q$ . This method can be expanded to include an oppositely charged point below the plane for each charged point above the plane in a more complex system. The systems suggested as thundercloud electrical models by Simpson and Robinson, Gish and Wait, and Malan are shown in Figure 10 using the method of electrical images.

Each of these models was derived using the data collected by the particular experimenter under a particular set of conditions. The models fit the individual data, yet they vary considerably in the overall field maps as shown in Figures 5, 6 and 7. For example,

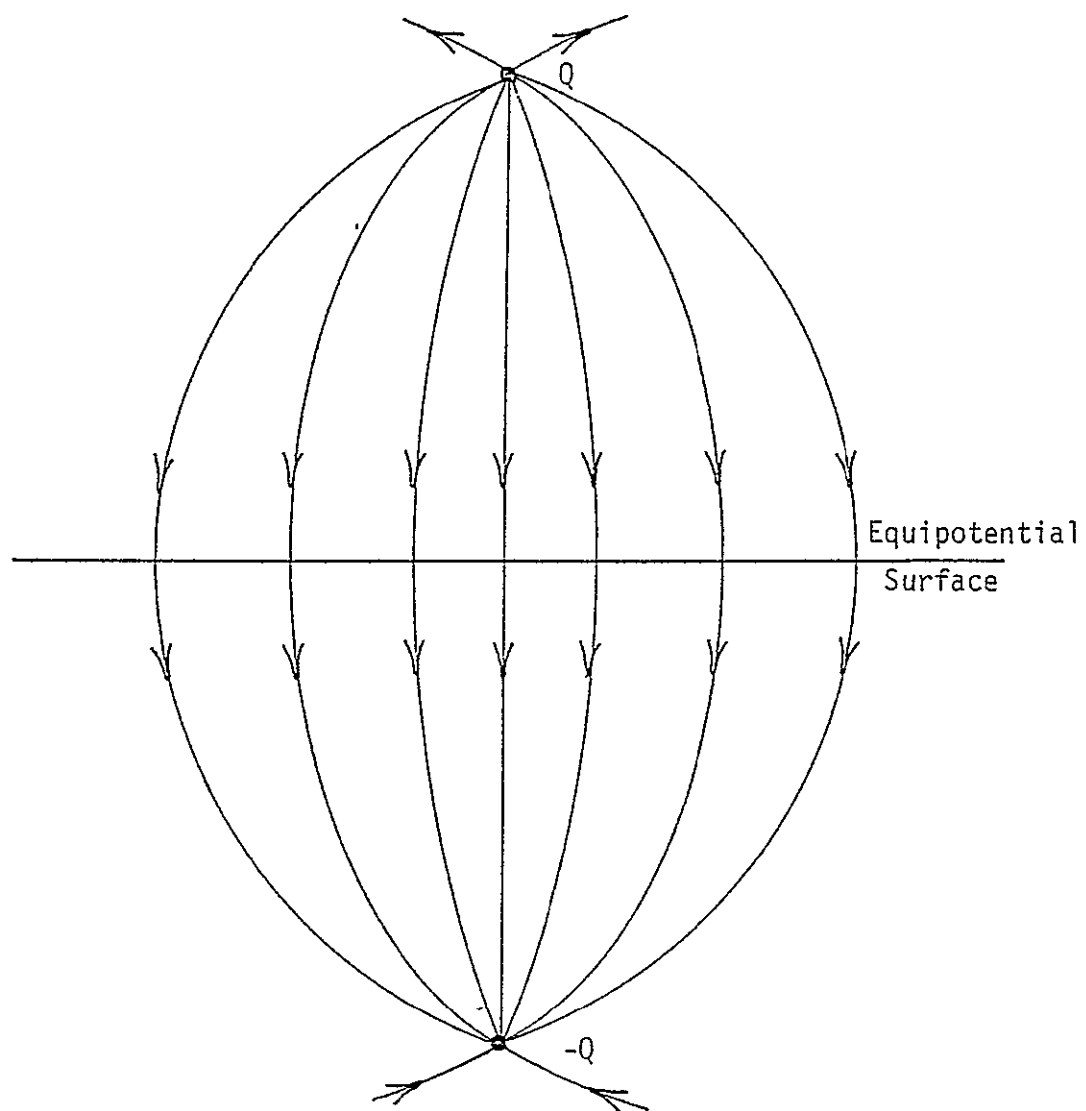
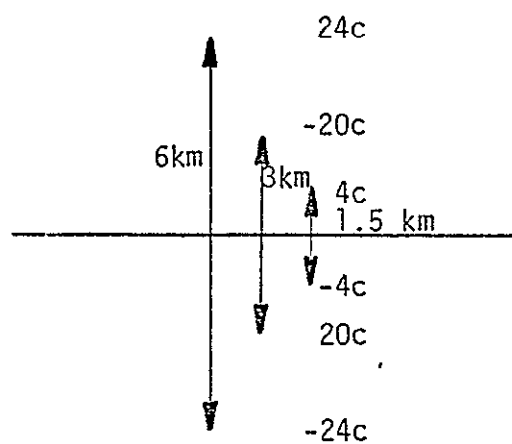
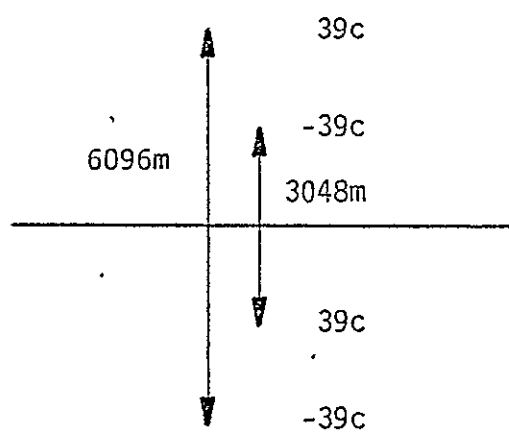


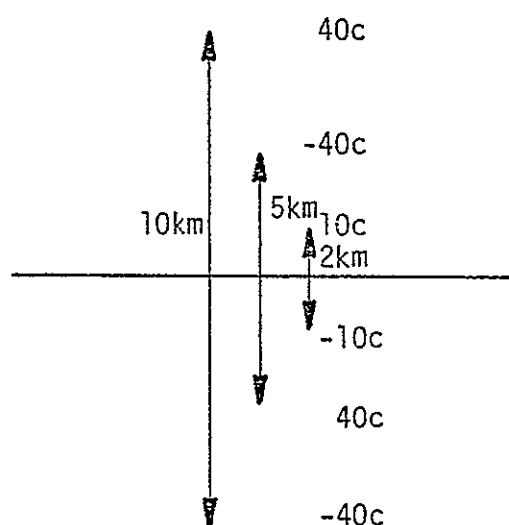
Figure 9



SIMPSON AND ROBINSON  
MODEL



GISH AND WATT  
MODEL



MALAN MODEL

Figure 10

at 500 meters horizontally from the charge centers and at ground level, the Simpson and Robinson model yields a field with a magnitude of 938 volt/meter, the Gish and Wait model yields a magnitude of 26122 volts/meter, while Malan's model has a magnitude of 19991 volts/meter at that location. If a model could be generated to best fit a combination of various field readings taken at different heights and different times under a variety of conditions, the result would be a more universal model of a thundercloud.

The model developed in this paper is found by starting with the charge center values and locations determined by Simpson and Robinson, Gish and Wait, and Malan. The field strength is calculated for selected locations using one model at a time and compared to actual values measured by various experimentors. The measured values used for the comparison and the source of each value are shown in Table 4. The readings from Wormell [10] are average potential gradients taken over all directions from the charge centers at ground level immediately before a lightning discharge.

The calculated value of the field using Equation 7 at location I,  $E_{FLD}(I)$ , is subtracted from the measured value,  $EF(I)$ , and the percentage of variation computed as

$$\frac{EF(I) - E_{FLD}(I)}{EF(I)} \times 100 \quad (8)$$

DISTANCE FROM CHARGE CENTERS (METERS)		MEASURED FIELD STRENGTH* (VOLTS/M)	SOURCE
1	2500	- 1310	Wormell [10]
2	5000	- 1160	"
3	7500	- 760	"
4	10000	- 460	"
5	12500	- 260	"
6	15000	- 135	"
7	17500	- 60	"
8	20000	0	"
9	13100	33000	Gish & Wait [ 8]

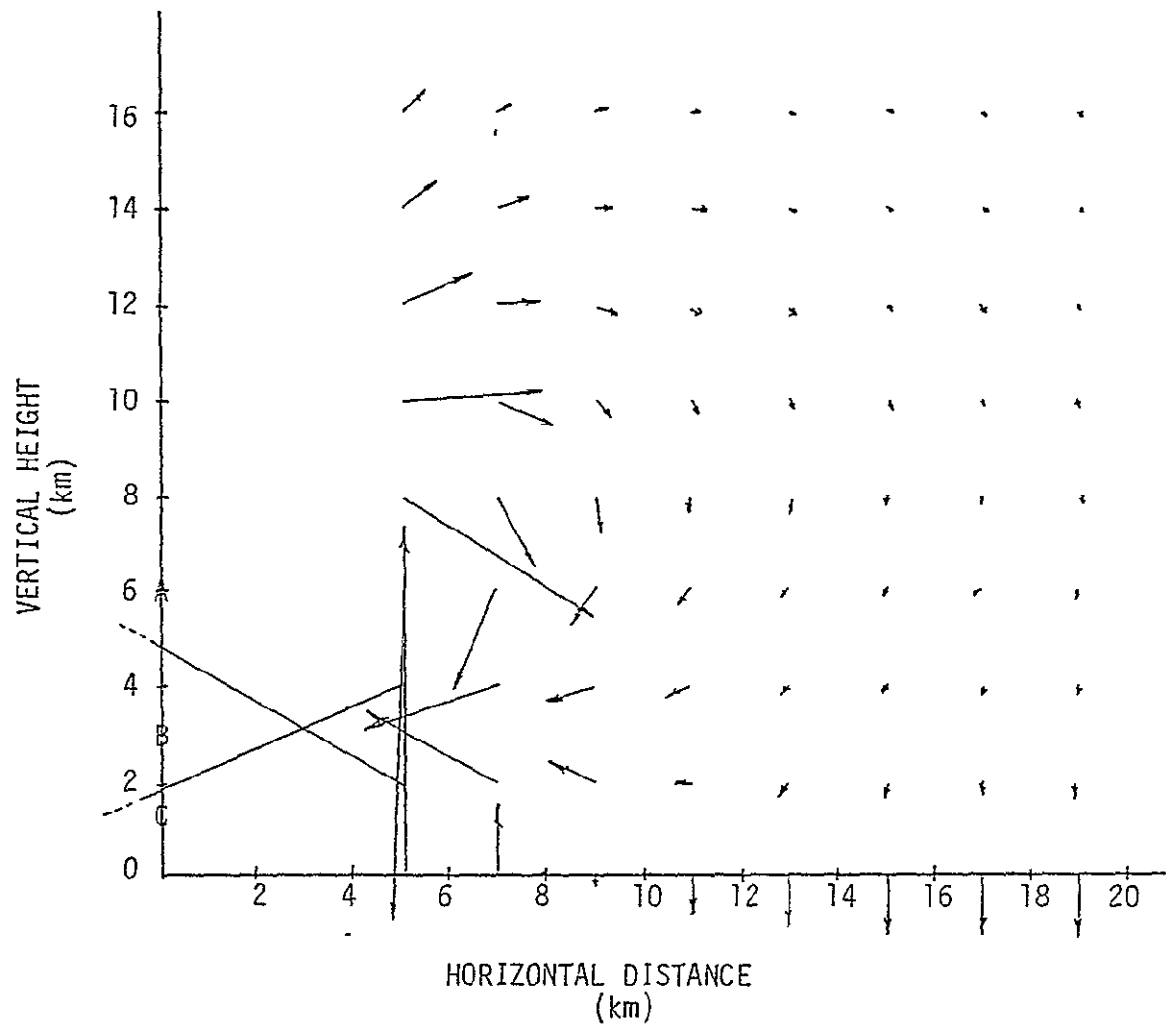
Note: 1 through 8 are horizontal distance with measurements made at ground level. 9 is a vertical height directly above the charge centers.

\*due to cloud only.

Table 4

The resulting value is then squared. The square of the percentage of variation is computed for each location in Table 4 and added together to produce a quantity ESQRT. The amount of charge at one of the charge centers is now varied in increments of .01 coulombs until a minimum is found for ESQRT. The process of calculating the fields and computing ESQRT is begun again with this new charge quantity replacing the old value for the selected charge center and a different charge center quantity is varied until a new minimum is found for ESQRT. The program continues to vary one charge center quantity at a time, each time finding the minimum ESQRT, until ESQRT is less than or equal to 100 or until ESQRT reaches a local minimum. The program listing and resulting output from the program are contained in Appendix B.

The resulting field maps of the Simpson and Robinson; Gish and Wait, and Malan models, after being modified by the program shown in Appendix B, are shown in Figures 11, 12 and 13 respectively. The progressive evaluation of the charge center values are shown in the output listings in Appendix B. The charge center values used for the modified Simpson and Robinson model are 4.6 coulombs at 6km, -11.3 coulombs at 3km and 5.97 coulombs at 1.5km. ESQRT had a value of  $5.22 \times 10^2$  at these values. The charge centers for the modified Gish and Wait model are 1.52 coulombs at 6096 meters and -2.64 coulombs at 3048 meters; ESQRT equals  $6.24 \times 10^2$ . The charge centers for the modified Malan model are 20.1 coulombs, -36.8 coulombs and 12.7 coulombs at 10km, 5km and 2km respectively; ESQRT equals  $7.99 \times 10^2$  at this point.

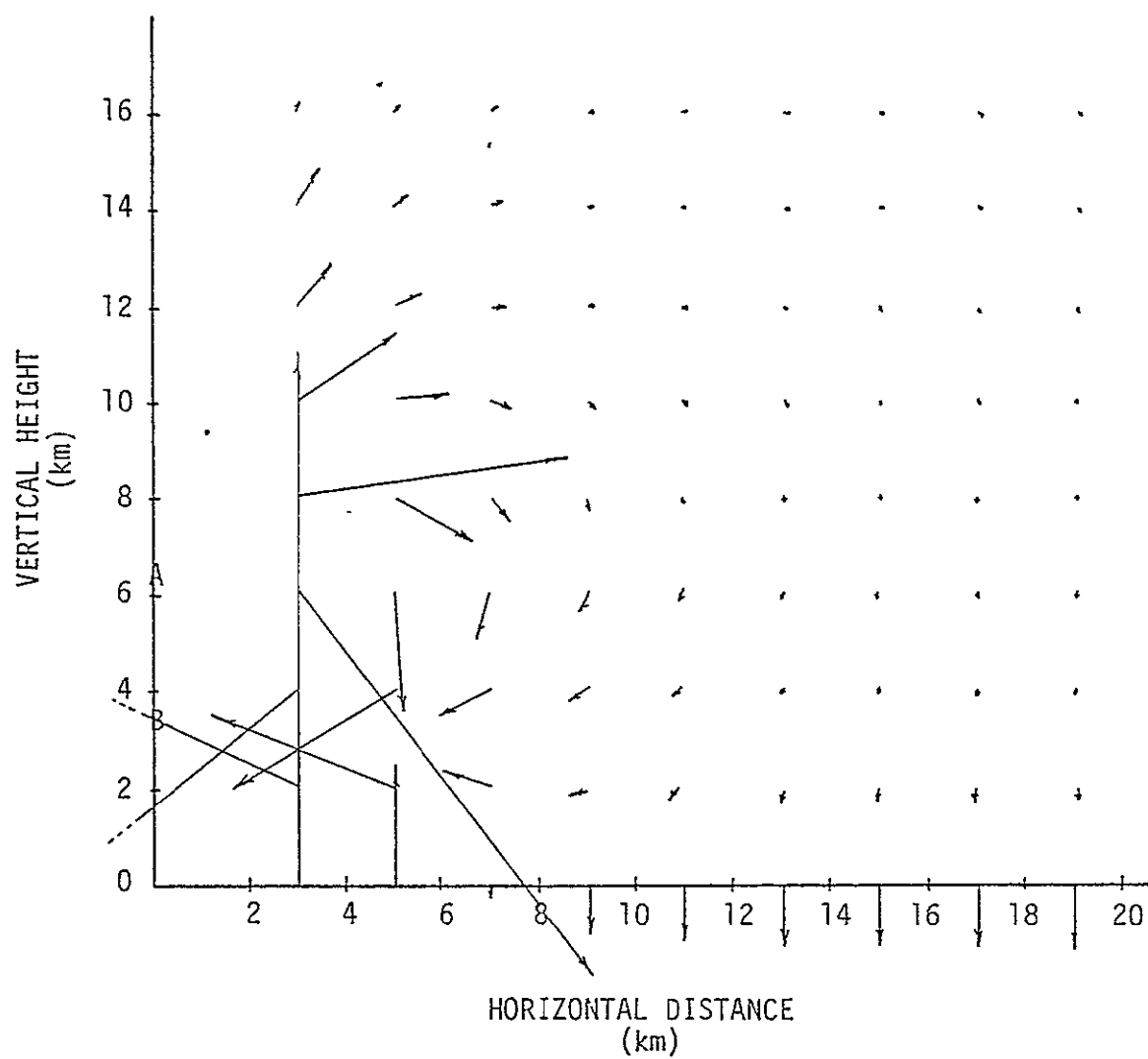
$1 \text{ in.} = 400 \text{ v/m}$ 

A = 4.6 coulombs  
B = -11.3 coulombs  
C = 5.97 coulombs

ELECTRIC FIELD MAP FOR  
MODIFIED SIMPSON AND ROBINSON MODEL

Figure 11

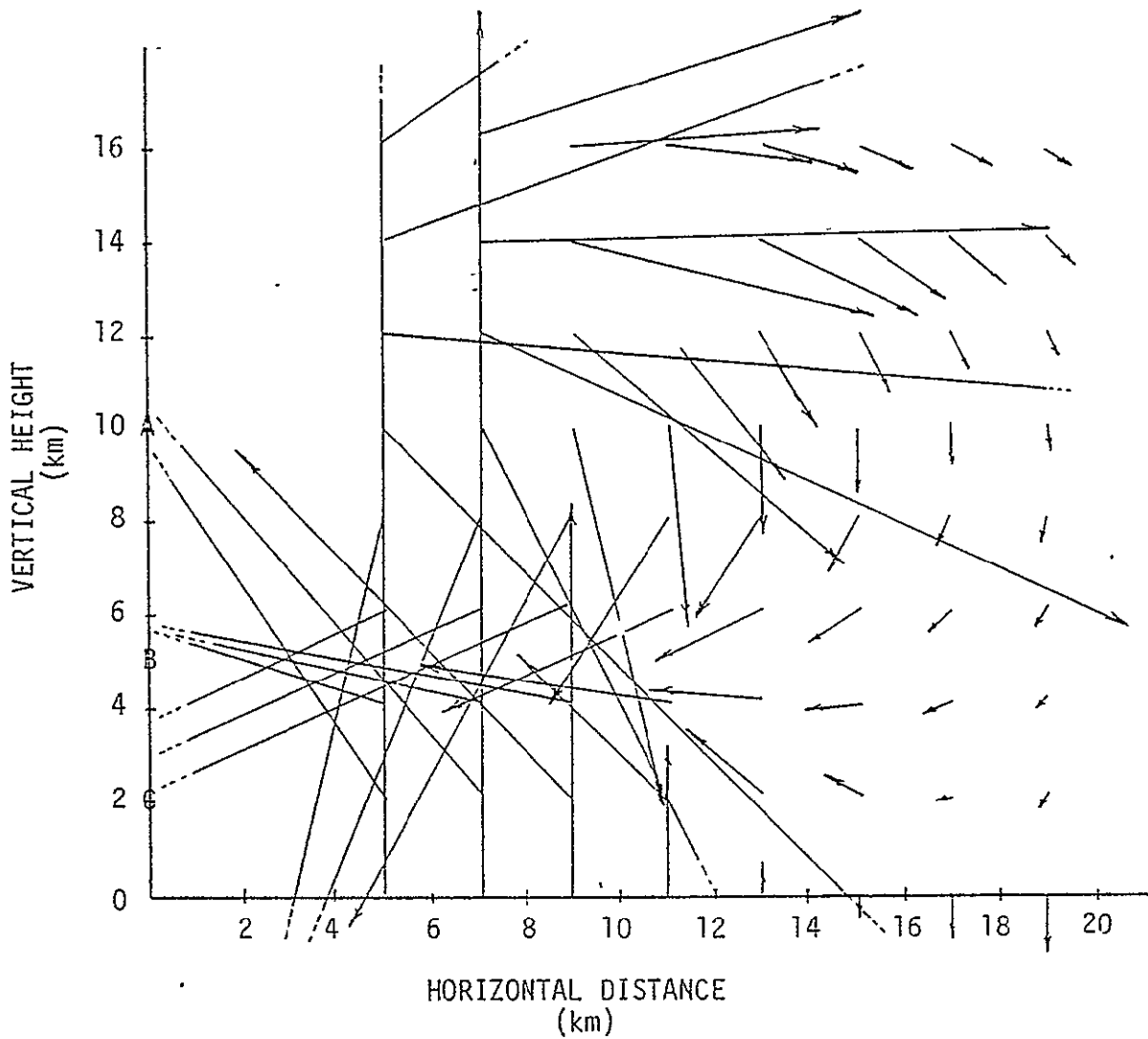


$I_n = 400 \text{ v/m}$ 

A = 1.52 coulombs  
B = -2.64 coulombs

ELECTRIC FIELD MAP FOR  
MODIFIED GISH AND WAIT MODEL

Figure 12

$1 \text{ In.} = 400 \text{ v/m}$ 

A = 20.1 coulombs  
B = -36.8 coulombs  
C = 12.7 coulombs

ELECTRIC FIELD MAP FOR  
MODIFIED MALAN MODEL

Figure 13

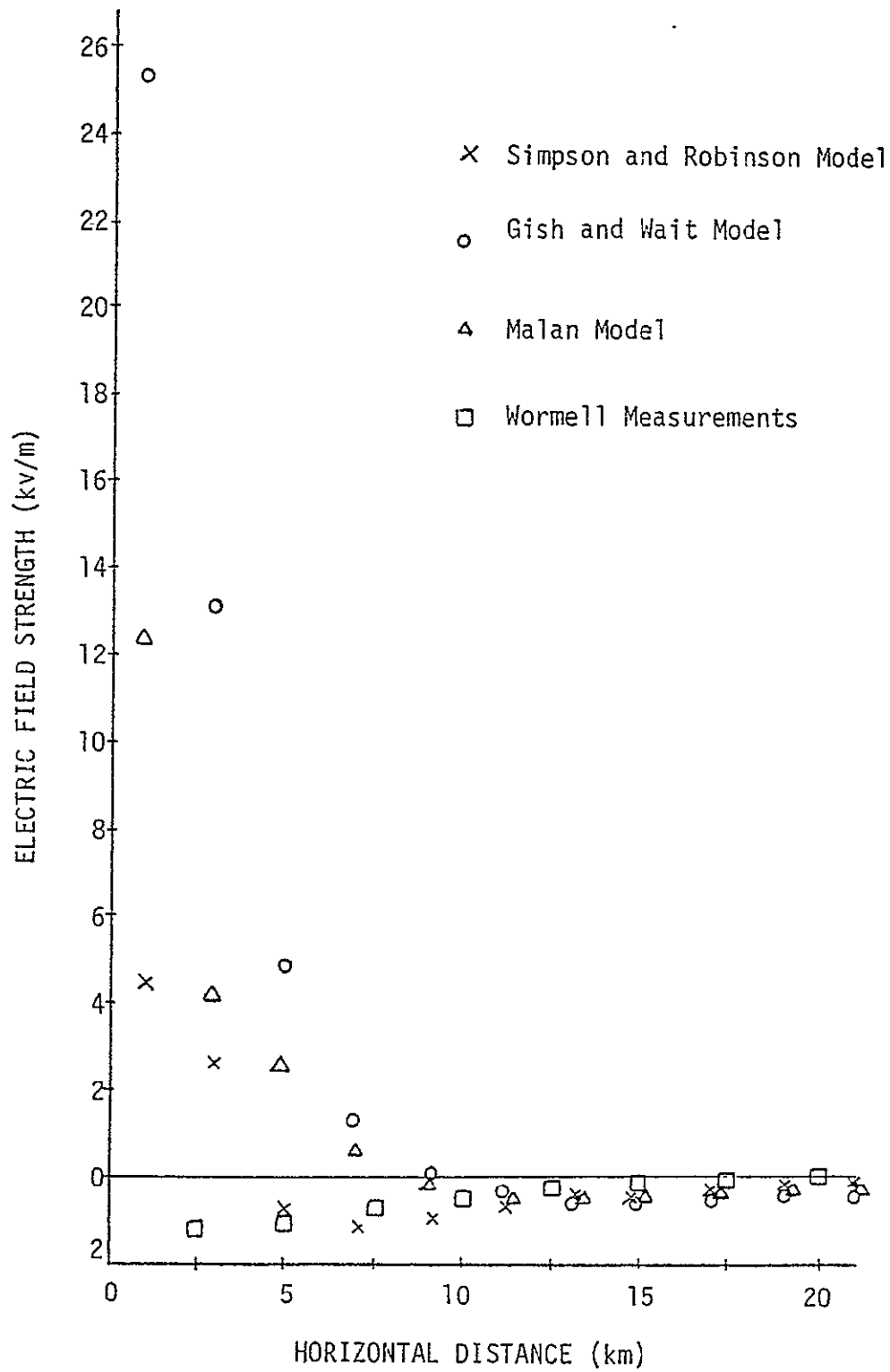
## VI. CONCLUSION

In order to select the model which most closely approximates a typical thundercloud, one must use more than the values of ESQRT which were generated. The modified models come closer to fitting the electric field values measured at ground level by Wormell and the values Gish and Wait measured to 40,000 feet than the original models as shown in Figures 14 and 15, yet the final model should most nearly conform to all electric field measurements made in the vicinity of a thundercloud without lightning present.

R. Markson [11] states that thunderclouds, not considering lightning flashes, can effect the electric field within 50km or more. In order to evaluate the ability of the modified models to meet this requirement, one can note the effect the various models have on the electric field at the farthest distance evaluated, i.e. 21km. Using an expression for the approximate electric field given by Dolezalek [1],  $h$  being the height in km,

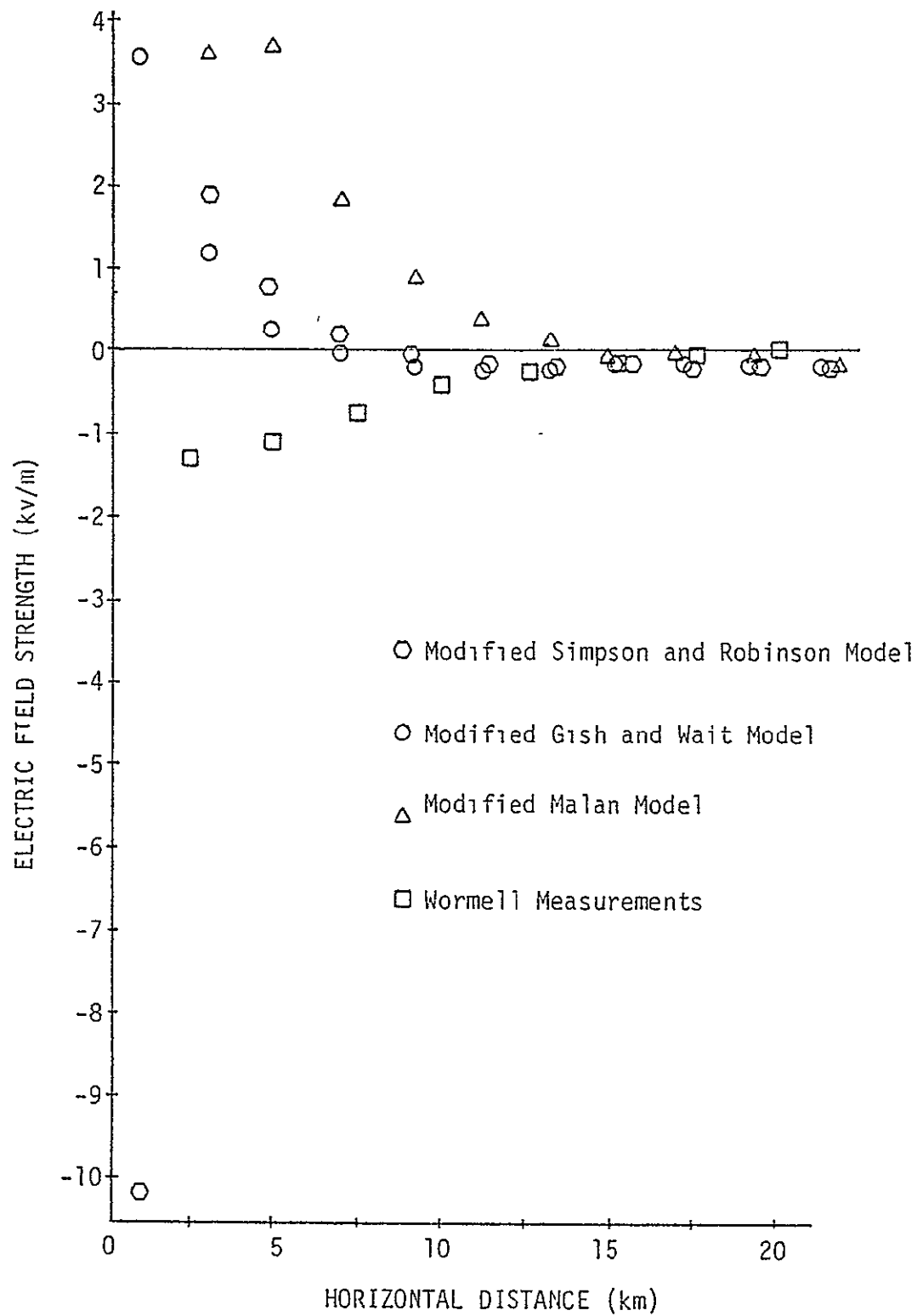
$$E = 81.8 \exp(-4.52h) + 38.6 \exp(-.375h) + 10.27 \exp(-.121h) \quad (9)$$

the field values at 4km, 10km and 16km are 4.59 v/m, .640 v/m and .119 v/m respectively. The values at 21km horizontal distance from the charge centers at these heights are 16.53 v/m at 73° from the vertical, 6.77 v/m at -12° from vertical and 4.24 v/m at -47° from vertical for the modified Simpson and Robinson model. The modified Gish and Wait model yields 15.8 v/m at -0.1°, 5.18 v/m at -10° and 2.53 v/m at -36° and



ELECTRIC FIELD STRENGTH AT GROUND LEVEL FOR  
ORIGINAL MODELS

Figure 14



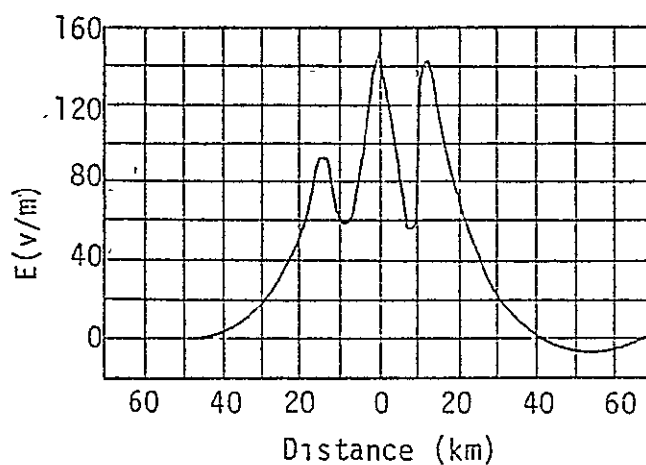
ELECTRIC FIELD STRENGTH AT GROUND LEVEL FOR THE  
MODIFIED MODELS

Figure 15

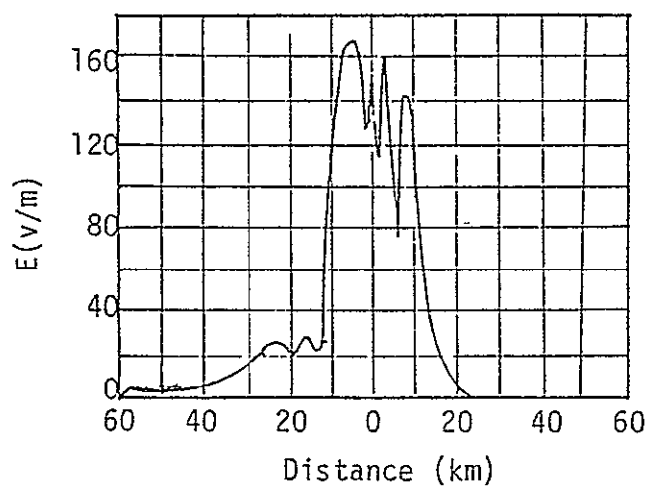
34.42 v/m at  $20^\circ$ , 45.2 v/m at  $-12^\circ$  and 51.15 v/m at  $-56^\circ$  for the modified Malan model. Since these values are for 21km and the field is to be effected within 50km or more, the largest resultant field values at the greatest angles must be chosen as being the closest fit. Here the modified Malan model is the best fit, followed by the modified Simpson and Robinson model, then the modified Gish and Wait model.

D.R. Lane-Smith [12] states that "tropical thunderstorms have been observed to produce negative electric fields of 3000 v/m or higher." All three modified models qualify under this criterion, Simpson and Robinson having a negative field as high as 47,000 v/m at a distance and height of 1km and 4 km, Gish and Wait producing 13,000 v/m at 1km and 4km height, and Malan producing 179,000 v/m at 1km and 10km height.

C.G. Stergis, G.C. Kein and T. Kangas [13] in balloon flights over thunderclouds made the measurements shown in Figures 16 and 17. The electric field measurements at a horizontal distance of 1km and a height of 16km for the three modified models are 85.17 v/m for Simpson and Robinson, 33.05 v/m for Gish and Wait and 2886.61 v/m for Malan. Because the value of the field strength will decrease as the height increases to that of the readings in Figures 16 and 17, i.e. above 21 km, the modified Malan model is the only one which could possibly satisfy these readings.



Flight at an altitude of 70,000 Feet  
on 21 July 1956

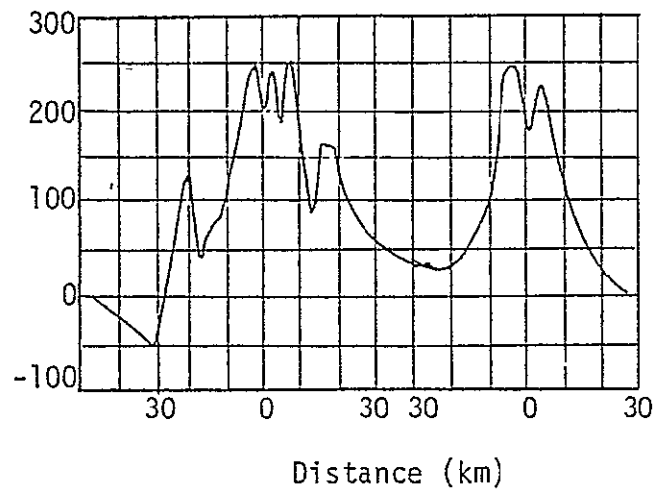


Flight at an altitude of 71,000 Feet  
on 30 July 1956

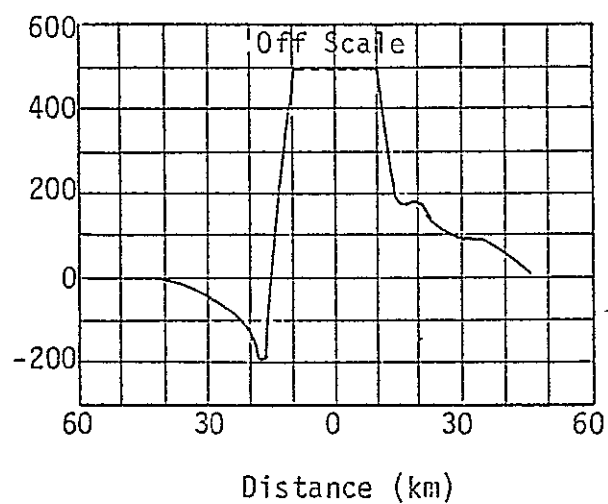
ELECTRIC FIELD MEASUREMENTS ABOVE  
THUNDERCLOUDS

Figure 16

(from Stergis, Rein and Kangas [13])



Flight at an altitude of 76,000 Feet  
on 3 August 1956



Flight at an altitude of 76,000 Feet  
on 20 July 1956

ELECTRIC FIELD MEASUREMENTS ABOVE  
THUNDERCLOUDS

Figure 17  
(from Stergis, Rein and Kangas [13])



Height above the thundercloud where E was measured,	0-100	100-300	300-500	500-1000	1,000
$E_{\max}$ , v/m	13,300	3,700	3,400	2,000	2,500
Number of measurements	34	29	40	24	13

DEPENDENCE OF  $E_{\max}$  ON HEIGHT ABOVE THUNDERCLOUD

Table 5  
(From Shvarts [14])

Similarly, Ya. M. Shvarts [14] gives the mean values of the maximum vertical component of the electric field at various distances above thunderclouds as shown in Table 5. In the modified Simpson and Robinson model the highest charge center is at 6km, the top of the cloud could therefore be considered to be somewhat below 8km. The vertical component of the electric field reading at 1km horizontal distance and the 8km height is 4821 v/m and, therefore, within the range of values in Table 5. The modified Gish and Wait model has its highest charge center also near 6km. In this case the vertical component of the electric field reading at 1km and 8km height is 1822 v/m. This is somewhat lower than the values in Table 5. The uppermost charge center of the modified Malan model is at 10km. The top of the cloud could be considered to be near 13km. Therefore, using the reading at 1km and 14km height, the vertical component is 7239 v/m. This is slightly high yet still within the range shown by Shvarts.

When considering the electrical specifications for a typical thunderstorm as described in the last two sections, the modified Malan model appears to be the best fit. The new electrical model of a cumulonimbus cloud has a charge of 20.1 coulombs at 10km, -36.8 coulombs at 5km and 12.7 coulombs at 2km.

The knowledge of the electric field surrounding a typical thundercloud has some practical uses and some limitations. Low cost avionics such as the field measuring device as described by M.L. Hill [14] and later by R. Markson [11] would be able to detect a field mapping of a thundercloud and therefore warn the pilot to avoid such an area. Yet, this model is only of a typical thundercloud; any specific thundercloud could exhibit quite different electric field characteristics, particularly during lightning, which has not been accounted for in this model. A model which would include lightning would demand an exceedingly complex system of which this model is only a beginning.

## APPENDIX A

The following program, written for the HP2100 computer, is used to produce a field map for a system of three charge centers of values  $Q(1)$ ,  $Q(2)$  and  $Q(3)$  at the corresponding heights of  $Y(1)$ ,  $Y(2)$  and  $Y(3)$ . The output lists the  $X$  and  $Y$  locations,  $X$  field value,  $Y$  field value, total field and the angle in radians of the total field at the location  $(X,Y)$ .  $X$  is incremented by 2000m from 1000m to 21000m;  $Y$  is incremented by 2000m from ground level to 16000m.

```

PROGRAM EMAP
REAL NX,NY,KNY
DIMENSION Y(6), Q(6)
20 READ (5,1) Y(1),Y(2),Y(3),Q(1),Q(2),Q(3)
1 FORMAT (6F10.2)
IF (Y(1)) 22,22,21
21 Y(4)=-Y(3)
Y(5)=-Y(2)
Y(6)=-Y(1)
Q(4)=-Q(3)
Q(5)=-Q(2)
Q(6)=-Q(1)
READ (5,4) A1,A2,A3,A4,A5,A6,A7
4 FORMAT (7A4)
WRITE (6,5) A1,A2,A3,A4,A5,A6,A7
5 FORMAT (1H1,45X,7A4,/,3X,"CHARGE CENTER 1",5X,"CHARGE CENTER 2",5
1X,"CHARGE CENTER 3",/,2X,"HEIGHT",3X,"COULOMBS",3X,"HEIGHT",3X,"CO
2ULOMBS",3X,"HEIGHT",3X,"COULOMBS")
WRITE (6,2) Y(1),Q(1),Y(2),Q(2),Y(3),Q(3)
2 FORMAT (1H,6F10.2)
WRITE (6,6)
6 FORMAT (/,3X,"X",5X,"Y",4X,"X FIELD",3X,"Y FIELD",4X,"TOTAL",5X,"
1THETA")
CONST = 4.*3.1416*8.854*(10.**(-12))
DO 10 NNY=0,16,2
NY=NNY*1000.
DO 10 NNX=10,210,20
NX=NNX*100.
EXSUM=0.
EYSUM=0.
DO 11 N=1,6,1
EXSUM=EXSUM +Q(N)*NX/(CONST*(NX**2+(NY-Y(N))*(NY-Y(N)))*1.5)
11 EYSUM=EYSUM+Q(N)*(NY-Y(N))/(CONST*(NX**2+(NY-Y(N))*(NY-Y(N)))*1.5
1)
KNY=NY/1000.
EYSUM=EYSUM-81.8*EXP(-4.52*KNY)-38.*EXP(-.375*KNY)-10.27*EXP(- 121
1*KNY)
IF (EXSUM) 15,13,12
12 THETA=ATAN(EYSUM/EXSUM)
16 ETOTL=SQRT(EXSUM*EXSUM+EYSUM*EYSUM)
14 WRITE (6,3) NX,NY,EXSUM,EYSUM,ETOTL,THETA
3 FORMAT (216,3F10.2,F10.4)
10 CONTINUE
GO TO 20
15 PI=3.1416
ANGLE=SIGN(PI,EYSUM)
THETA=ANGLE+ATAN(EYSUM/EXSUM)
GO TO 16
13 ANGLE=3.1416/2.
THETA=SIGN(ANGLE,EYSUM)
ETOTL=EYSUM
GO TO 14
22 END

```

## SIMPSON AND ROBINSON MODEL

CHARGE CENTER 1		CHARGE CENTER 2		CHARGE CENTER 3	
HEIGHT	COULOMBS	HEIGHT	COULOMBS	HEIGHT	COULOMBS
6000.00	24.00	3000.00	-20.00	1500.00	4.00
X	Y	X FIELD	Y FIELD	TOTAL	THETA
1000	0	.00	4066.83	4066.83	1.5708
3000	0	.00	2559.72	2559.72	1.5708
5000	0	.00	-881.20	-881.20	-1.5708
7000	0	.00	-1285.40	-1285.40	-1.5708
9000	0	.00	-1054.19	-1054.19	-1.5708
11000	0	.00	-797.06	-797.06	-1.5708
13000	0	.00	-605.92	-605.92	-1.5708
15000	0	.00	-474.12	-474.12	-1.5708
17000	0	.00	-383.90	-383.90	-1.5708
19000	0	.00	-321.34	-321.34	-1.5708
21000	0	.00	-277.11	-277.11	-1.5708
1000	2000	-34552.17	64956.74	73574.67	2.0597
3000	2000	-7460.80	-123.11	7451.82	-3.1251
5000	2000	-787.46	-1881.66	2039.79	-1.9671
7000	2000	245.10	-1399.23	1420.54	-1.3974
9000	2000	306.39	-967.06	1014.43	-1.2640
11000	2000	229.31	-675.36	713.22	-1.2435
13000	2000	157.24	-483.80	508.71	-1.2566
15000	2000	106.70	-356.71	372.33	-1.2801
17000	2000	73.30	-270.68	280.43	-1.3063
19000	2000	51.36	-211.07	217.23	-1.3321
21000	2000	36.75	-168.81	172.76	-1.3564
1000	4000	-42327.79	-97247.95	106060.39	-1.9813
3000	4000	-1222.37	-13244.89	13301.17	-1.6628
5000	4000	1358.17	-3666.89	3910.34	-1.2161
7000	4000	1077.23	-1591.73	1921.99	-.9758
9000	4000	714.07	-902.53	1150.84	-.9015
11000	4000	463.20	-589.46	749.68	-.9048
13000	4000	303.79	-415.05	514.35	-.9390
15000	4000	203.43	-305.87	367.34	-.9839
17000	4000	139.51	-232.83	271.43	-1.0310
19000	4000	97.98	-181.91	206.62	-1.0767
21000	4000	70.38	-145.35	161.49	-1.1199
1000	6000	210423.44	-15336.63	210981.59	-.0728
3000	6000	17672.71	-6032.24	18673.86	-.3289
5000	6000	4772.50	-2261.52	5281.21	-.4425
7000	6000	2042.89	-1072.73	2307.41	-.4835

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
9000	6000	1093.99	-631.81	1263.32	-.5237
11000	6000	655.36	-427.21	782.31	-.5777
13000	6000	418.42	-311.83	521.84	-.6405
15000	6000	278.75	-237.61	366.28	-.7059
17000	6000	191.85	-186.06	267.25	-.7701
19000	6000	135.65	-148.64	201.24	-.8310
21000	6000	98.18	-120.74	155.62	-.8881
1000	8000	18077.88	32606.24	37282.39	1.0646
3000	8000	11414.12	5261.37	12568.38	.4319
5000	8000	4725.81	563.86	4759.33	.1188
7000	8000	2246.16	-134.77	2250.20	-.0599
9000	8000	1229.38	-226.77	1250.12	-.1824
11000	8000	742.58	-215.60	773.25	-.2826
13000	8000	478.86	-187.20	514.15	-.3727
15000	8000	323.02	-158.48	359.85	-.4564
17000	8000	225.34	-133.35	261.84	-.5343
19000	8000	161.45	-112.08	196.54	-.6068
21000	8000	118.28	-94.50	151.40	-.6741
1000	10000	2631.71	9182.00	9551.70	1.2917
3000	10000	4116.18	4404.92	6028.78	.8193
5000	10000	2896.25	1547.05	3283.54	.4906
7000	10000	1777.12	483.03	1841.60	.2654
9000	10000	1102.76	111.90	1108.42	.1011
11000	10000	712.94	-20.71	713.24	-.0290
13000	10000	479.75	-67.37	484.45	-.1395
15000	10000	333.74	-80.73	343.37	-.2373
17000	10000	238.55	-80.62	251.80	-.3259
19000	10000	174.38	-75.18	189.90	-.4071
21000	10000	129.96	-67.94	146.64	-.4817
1000	12000	748.58	3825.96	3898.50	1.3776
3000	12000	1598.30	2610.45	3060.89	1.0214
5000	12000	1556.23	1395.26	2090.12	.7309
7000	12000	1199.36	662.58	1370.21	.5047
9000	12000	860.01	291.66	908.12	.3270
11000	12000	609.46	112.35	619.73	.1823
13000	12000	435.98	26.07	436.76	.0597
15000	12000	316.79	-15.06	317.14	-.0475
17000	12000	234.00	-33.81	236.43	-.1435
19000	12000	175.57	-41.23	180.35	-.2307
21000	12000	133.65	-42.89	140.36	-.3106
1000	14000	296.39	1984.18	2006.20	1.4225
3000	14000	723.55	1573.04	1731.51	1.1396
5000	14000	845.55	1044.32	1343.71	.8902
7000	14000	767.55	620.30	986.87	.6797
9000	14000	623.99	344.12	712.58	.5040
11000	14000	484.45	179.98	516.81	.3557

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
13000	14000	370.29	85.95	380.14	.2281
15000	14000	282.64	32.89	284.55	.1159
17000	14000	216.82	3.36	216.84	.0155
19000	14000	167.62	-12.68	168.10	-.0755
21000	14000	130.74	-20.94	132.41	-.1588
1000	16000	142.35	1173.79	1182.39	1.4501
3000	16000	372.18	1004.45	1071.18	1.2160
5000	16000	483.71	754.30	896.07	1.0006
7000	16000	490.83	515.07	711.48	.8095
9000	16000	440.14	329.57	549.85	.6427
11000	16000	370.05	200.88	421.06	.4973
13000	16000	301.17	116.66	322.98	.3696
15000	16000	241.51	63.21	249.64	.2560
17000	16000	192.67	29.86	194.97	.1537
19000	16000	153.74	9.36	154.03	.0608
21000	16000	123.08	-3.00	123.12	-.0244

GISH AND WAIT MODEL

CHARGE CENTER 1		CHARGE CENTER 2		CHARGE CENTER 3	
HEIGHT	COULOMBS	HEIGHT	COULOMBS	HEIGHT	COULOMBS
6096.00	39.00	3048.00	-39.00	.00	.00
X	Y	X FIELD	Y FIELD	TOTAL	THETA
1000	0	-.00	46473.19	46473.19	1.5708
3000	0	.00	13560.42	13560.42	1.5708
5000	0	-.00	1791.56	1791.56	1.5708
7000	0	.00	-672.18	-672.18	-1.5708
9000	0	.00	-966.69	-966.69	-1.5708
11000	0	.00	-841.81	-841.81	-1.5708
13000	0	.00	-676.21	-676.21	-1.5708
15000	0	.00	-541.02	-541.02	-1.5078
17000	0	.00	-440.81	-440.81	-1.5708
19000	0	-.00	-368.09	368.09	-1.5708
21000	0	.00	-315.17	-315.17	-1.5708
1000	2000	-108718.89	109432.34	154257.06	2.3529
3000	2000	-21174.68	4780.39	21707.59	2.9196
5000	2000	-3802.59	-948.36	3919.07	-2.8972
7000	2000	-504.58	-1243.51	1341.98	-1.9563
9000	2000	114.77	-1006.48	1013.00	-1.4573
11000	2000	187.85	-757.27	780.22	-1.3276
13000	2000	157.06	-565.07	586.50	-1.2997
15000	2000	116.66	-426.54	442.21	-1.3038
17000	2000	84.33	-328.00	338.67	-1.3191
19000	2000	61.00	-257.49	264.62	-1.3382
21000	2000	44.61	-206.39	211.15	-1.3579
1000	4000	-104554.09	-181997.62	209892.09	-2.0922
3000	4000	-10832.73	-23238.61	25639.44	-2.0070
5000	4000	-803.15	-5802.10	5857.43	-1.7084
7000	4000	509.48	-2232.55	2289.94	-1.3464
9000	4000	578.85	-1172.51	1307.61	-1.1122
11000	4000	452.19	-741.18	868.24	-1.0230
13000	4000	325.58	-516.84	610.84	-1.0087
15000	4000	230.37	-380.52	444.82	-1.0264
17000	4000	163.69	-290.07	333.07	-1.0570
19000	4000	117.79	-226.91	255.66	-1.0920
21000	4000	86.10	-181.33	200.73	-1.1275
1000	6000	334420.31	-65541.23	340782.31	-.1935
3000	6000	25453.36	-13660.82	28887.57	-.4926
5000	6000	5864.96	-4583.87	7443.76	-.6634



X	Y	X FIELD	Y FIELD	TOTAL	THETA
7000	6000	2295.37	-1901.85	2980.90	-.6919
9000	6000	1211.50	-984.07	1560.81	-.6822
11000	6000	738.83	-607.15	956.30	-.6879
13000	6000	483.46	-420.12	640.50	-.7154
15000	6000	329.47	-311.14	453.16	-.7568
17000	6000	231.05	-240.02	333.16	-.8044
19000	6000	165.83	-190.20	252.34	-.8537
21000	6000	121.47	-153.73	195.93	-.9021
1000	8000	32652.30	54711.66	63714.52	1.0327
3000	8000	18372.59	6858.57	19611.02	.3573
5000	8000	6873.42	65.88	6873.74	.0096
7000	8000	3002.36	-547.50	3051.88	-.1804
9000	8000	1557.87	-467.76	1626.58	-.2917
11000	8000	917.97	-357.29	985.05	-.3712
13000	8000	587.85	-277.19	649.92	-.4406
15000	8000	397.35	-220.69	454.52	-.5070
17000	8000	278.74	-179.23	331.39	-.5714
19000	8000	201.04	-147.64	249.43	-.6334
21000	8000	418.25	-122.96	192.61	-.6924
1000	10000	4416.47	14565.66	15220.50	1.2764
3000	10000	6586.64	6468.16	9231.52	.7763
5000	10000	4350.91	1974.11	4777.82	.4259
7000	10000	2512.04	481.38	2557.75	.1893
9000	10000	1485.50	37.40	1485.97	.0252
11000	10000	930.13	-90.61	934.53	-.0971
13000	10000	614.63	-122.40	626.70	-.1966
15000	10000	423.93	-123.41	441.53	-.2833
17000	10000	302.16	-114.40	323.09	-.3619
19000	10000	220.97	-102.57	243.62	-.4346
21000	10000	165.01	-90.60	188.25	-.5021
1000	12000	1196.20	5814.28	5936.05	1.3679
3000	12000	2491.73	3826.18	4566.00	.9936
5000	12000	2333.49	1919.59	3021.59	.6884
7000	12000	1724.09	839.38	1917.56	.4531
9000	12000	1190.52	332.74	1236.15	.2725
11000	12000	819.34	107.33	826.34	.1303
13000	12000	574.33	7.58	574.38	.0132
15000	12000	412.00	-36.05	413.58	-.0873
17000	12000	302.12	-53.86	306.89	-.1764
19000	12000	225.87	-59.41	233.56	-.2572
21000	12000	171.73	-59.08	181.61	-.3313
1000	14000	456.98	2925.56	2961.03	1.4158
3000	14000	1099.50	2273.60	2525.50	1.1204
5000	14000	1253.13	1456.99	1921.76	.8605
7000	14000	1104.81	827.48	1380.34	.6429
9000	14000	873.05	436.67	976.17	.4638

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
11000	14000	661.57	215.99	695.94	.3156
13000	14000	496.24	95.67	505.37	.1905
15000	14000	373.66	30.78	374.92	.0822
17000	14000	284.04	-3.87	284.06	-.0136
19000	14000	218.33	-21.90	219.43	-.1000
21000	14000	169.73	-30.69	172.48	-.1789
1000	16000	213.59	1693.36	1706.78	1.4453
3000	16000	553.23	1431.78	1534.94	1.2021
5000	16000	707.20	1052.13	1267.72	.9790
7000	16000	703.10	698.73	991.25	.7823
9000	16000	617.32	433.55	754.35	.6123
11000	16000	509.07	256.01	569.82	.4660
13000	16000	407.65	143.80	432.27	.3391
15000	16000	322.76	74.81	331.31	.2278
17000	16000	255.06	32.95	257.18	.1285
19000	16000	202.18	7.85	202.34	.0388
21000	16000	161.14	-6.94	161.28	-.0431

## MALAN MODEL

CHARGE CENTER 1		CHARGE CENTER 2		CHARGE CENTER 3	
HEIGHT	COULOMBS	HEIGHT	COULOMBS	HEIGHT	COULOMBS
10000.00	40.00	5000.00	-40.00	2000.00	10.00
X	Y	X FIELD	Y FIELD	TOTAL	THETA
1000	0	.00	-12251.73	12251.73	-1.5708
3000	0	-.00	4015.52	4015.52	1.5708
5000	0	.00	2591.42	2591.52	1.5708
7000	0	-.00	632.48	632.48	1.5708
9000	0	.00	-247.33	-247.33	-1.5708
11000	0	.00	-537.95	-537.95	-1.5708
13000	0	.00	-587.30	-587.30	-1.5708
15000	0	.00	-551.52	-551.52	-1.5708
17000	0	.00	-492.79	-492.79	-1.5708
19000	0	.00	-433.99	-433.99	-1.5708
21000	0	.00	-382.39	-382.39	-1.5708
1000	2000	78723.22	28109.79	83591.31	.3430
3000	2000	-2692.49	10027.17	10382.37	1.8331
5000	2000	-3037.11	2608.98	4003.85	2.4319
7000	2000	-1313.15	320.30	1351.65	2.9024
9000	2000	-449.35	-366.78	580.04	-2.4570
11000	2000	-102.73	-530.26	540.12	-1.7622
13000	2000	21.59	-520.62	521.07	-1.5294
15000	2000	58.41	-459.97	463.67	-1.4445
17000	2000	62.82	-391.33	396.34	-1.4116
19000	2000	56.43	-328.87	333.68	-1.4009
21000	2000	47.45	-276.16	280.20	-1.4006
1000	4000	-117514.45	133725.22	178022.72	2.2918
3000	4000	-24778.07	8331.59	26141.32	2.8172
5000	4000	-6751.43	-378.73	6762.04	-3.0856
7000	4000	-2037.31	-1102.32	2316.41	-2.6456
9000	4000	-540.37	-954.34	1096.70	-2.0860
11000	4000	-42.81	-754.78	756.00	-1.6275
13000	4000	111.29	-595.46	605.77	-1.3860
15000	4000	144.15	-474.77	496.17	-1.2760
17000	4000	135.96	-382.97	406.39	-1.2297
19000	4000	116.25	-312.38	333.31	-1.2145
21000	4000	95.57	-257.55	274.71	-1.2155
1000	6000	-120686.14	-142335.00	186612.97	-2.2741
3000	6000	-23275.31	-19824.02	30573.39	-2.4361
5000	6000	-4897.77	-6664.76	8270.87	-2.2046

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
7000	6000	-976.07	-2978.22	3134.08	-1.8875
9000	6000	16.83	-1586.22	1586.31	-1.5602
11000	6000	265.29	-969.45	1005.09	-1.3037
13000	6000	297.59	-657.70	721.90	-1.1459
15000	6000	266.77	-480.23	549.35	-1.0637
17000	6000	222.12	-368.41	430.91	-1.0282
19000	6000	179.59	-292.18	342.97	-1.0197
21000	6000	143.60	-237.19	277.27	-1.0264
1000	8000	21198.36	-95907.80	98222.58	-1.3533
3000	8000	9820.07	-27568.87	29265.62	-1.2286
5000	8000	3454.41	-8824.13	9475.19	-1.1977
7000	8000	1714.37	-3563.27	3954.23	-1.1224
9000	8000	1067.79	-1741.09	2042.45	-1.0207
11000	8000	742.80	-990.34	1237.95	-.9273
13000	8000	544.33	-634.98	836.36	-.8621
15000	8000	409.54	-445.56	604.19	-.8275
17000	8000	313.02	-333.47	457.35	-.8170
19000	8000	241.97	-260.99	355.90	-.8232
21000	8000	188.84	-210.66	282.91	-.8399
1000	10000	356977.94	-12116.47	357183.50	-.0339
3000	10000	34966.09	-7850.73	35836.60	-.2209
5000	10000	9876.40	-4179.14	10724.20	-.4003
7000	10000	3963.03	-2198.17	4531.83	-.5064
9000	10000	1994.93	-1232.97	2345.20	-.5536
11000	10000	1175.08	-753.90	1396.13	-.5704
13000	10000	768.44	-502.01	917.89	-.5787
15000	10000	537.57	-359.77	646.85	-.5898
17000	10000	392.89	-273.12	478.49	-.4076
19000	10000	295.74	-216.30	366.40	-.6315
21000	10000	227.34	-176.57	287.85	-.6604
1000	12000	31233.70	58117.42	65978.62	1.0777
3000	12000	20824.47	10466.50	23306.79	.4657
5000	12000	9037.53	1317.70	9133.19	.1448
7000	12000	4310.54	-266.50	4316.48	-.0525
9000	12000	2316.31	-417.18	2353.58	-.1782
11000	12000	1381.88	-368.39	1430.15	-.2605
13000	12000	895.93	-295.87	943.52	-.3190
15000	12000	618.29	-236.96	662.14	-.3650
17000	12000	446.61	-193.30	486.64	-.4085
19000	12000	333.48	-160.96	370.29	-.4497
21000	12000	255.22	-136.36	289.36	-.4907
1000	14000	4700.79	16794.50	17439.98	1.2979
3000	14000	7520.32	8299.14	11199.60	.8346

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
5000	14000	5424.10	3011.47	6204.01	.5068
7000	14000	3362.94	961.95	3497.82	.2786
9000	14000	2082.60	243.79	2096.82	.1165
11000	14000	1341.62	-2.14	1341.62	-.0016
13000	14000	905.85	-83.00	909.65	-.0914
15000	14000	638.60	-105.42	647.24	-.1636
17000	14000	466.70	-107.03	478.82	-.2254
19000	14000	351.07	-101.35	365.41	-.2810
21000	14000	270.20	-93.40	285.89	-.3328
1000	16000	1366.02	7108.68	7238.74	1.3809
3000	16000	2945.49	4908.01	5724.03	1.0303
5000	16000	2903.74	2666.07	3942.04	.7428
7000	16000	2255.76	1287.59	2597.37	.5187
9000	16000	1621.80	584.09	1723.78	.3457
11000	16000	1150.72	246.31	1176.79	.2109
13000	16000	826.67	85.77	831.11	.1034
15000	16000	606.64	9.07	606.71	.0150
17000	16000	455.31	-27.59	456.15	-.0605
19000	16000	348.89	-44.61	351.73	-.1272
21000	16000	272.20	-51.70	277.07	-.1877

## MODIFIED SIMPSON AND ROBINSON MODEL

CHARGE CENTER 1		CHARGE CENTER 2		CHARGE CENTER 3	
HEIGHT	COULOMBS	HEIGHT	COULOMBS	HEIGHT	COULOMBS
6000.00	4.60	3000.00	-11.30	1500.00	5.97

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
1000	0	.00	-10538.48	-10538.48	-1.5708
3000	0	.00	1939.86	1939.85	1.5708
5000	0	-.00	770.68	770.68	1.5708
7000	0	-.00	177.66	177.66	1.5708
9000	0	.00	-20.31	-20.31	-1.5708
11000	0	-.00	-88.79	88.79	-1.5708
13000	0	-.00	-114.33	114.33	-1.5708
15000	0	-.00	-124.48	124.48	-1.5708
17000	0	.00	-218.68	-128.68	-1.5708
19000	0	.00	-130.42	-130.42	-1.5708
21000	0	-.00	-131.11	131.11	-1.5708
1000	2000	2650.72	52024.13	52091.62	1.5199
3000	2000	-3226.18	2930.23	4358.27	2.4042
5000	2000	-918.40	537.89	1064.33	2.6118
7000	2000	-279.52	153.89	319.08	2.6383
9000	2000	-93.95	41.60	102.75	2.7247
11000	2000	-34.56	.63	34.56	3.1235
13000	2000	-13.63	-15.77	20.84	-2.2837
15000	2000	-5.63	-22.64	23.33	-1.8145
17000	2000	-2.35	-25.57	25.68	-1.6630
19000	2000	-.96	-26.79	26.81	-1.6066
21000	2000	-.34	-27.25	27.25	-1.5832
1000	4000	-29521.43	-36532.12	46969.25	-2.2505
3000	4000	-4359.65	-2692.04	5123.83	-2.5884
5000	4000	974.39	-440.09	1069.16	-2.7174
7000	4000	-304.88	-102.55	321.66	-2.8171
9000	4000	-111.29	-34.15	116.41	-2.8439
11000	4000	-44.17	-19.65	48.34	-2.7231
13000	4000	-18.43	-16.91	25.02	-2.3993
15000	4000	-7.89	-16.60	18.38	-2.0143
17000	4000	-3.35	-16.64	16.97	-1.7693
19000	4000	-1.32	-16.62	16.67	-1.6502
21000	4000	-.41	-16.52	16.53	-1.5956
1000	6000	38668.84	-7161.18	39326.34	-.1831
3000	6000	1607.50	-2427.45	2911.46	-.9859
5000	6000	-21.39	-690.32	690.65	-1.6018
7000	6000	-90.12	-220.81	238.49	-1.9583
9000	6000	-53.42	-82.80	98.54	-2.1438

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
11000	6000	-26.85	-37.86	46.42	-2.1877
13000	6000	-12.78	-21.92	25.37	-2.0985
15000	6000	-5.82	-15.78	16.81	-1.9240
17000	6000	-2.44	-13.17	13.39	-1.7544
19000	6000	-.83	-11.92	11.95	-1.6404
21000	6000	-.08	-11.23	11.23	-1.5778
1000	8000	3119.30	4820.74	5741.91	.9965
3000	8000	1548.54	188.47	1559.96	.1211
5000	8000	381.93	-235.11	448.49	-.5518
7000	8000	80.84	-145.22	167.08	-1.0658
9000	8000	12.37	-74.57	75.59	-1.4065
11000	8000	-.95	-39.09	39.10	-1.5951
13000	8000	-2.06	-22.69	22.79	-1.6615
15000	8000	-1.10	-15.06	15.10	-1.6439
17000	8000	-.20	-11.36	11.36	-1.5885
19000	8000	.34	-9.46	9.47	-1.5350
21000	8000	.60	-8.41	8.43	-1.5000
1000	10000	389.07	1106.02	1172.46	1.2325
3000	10000	525.92	367.35	641.52	.6097
5000	10000	277.42	19.99	278.14	.0719
7000	10000	113.39	-46.45	122.53	-.3888
9000	10000	43.38	-41.14	59.79	-.7589
11000	10000	16.93	-27.92	32.65	-1.0257
13000	10000	7.27	-18.44	19.82	-1.1950
15000	10000	3.73	-12.78	13.31	-1.2868
17000	10000	2.36	-9.55	9.84	-1.3285
19000	10000	1.76	-7.71	7.90	-1.3462
21000	10000	1.43	-6.62	6.77	-1.3583
1000	12000	93.86	377.89	389.37	1.3274
3000	12000	181.48	210.26	277.75	.8587
5000	12000	145.99	66.95	160.61	.4300
7000	12000	86.33	3.99	86.42	.0462
9000	12000	45.34	-13.40	47.28	-.2873
11000	12000	23.26	-14.64	27.48	-.5617
13000	12000	12.34	-12.05	17.24	-.7734
15000	12000	7.04	-9.38	11.72	-.9270
17000	12000	4.40	-7.41	8.61	-1.0351
19000	12000	3.00	-6.08	6.78	-1.1119
21000	12000	2.21	-5.22	5.66	-1.1707
1000	14000	31.83	164.82	167.86	1.3800
3000	14000	72.40	114.53	135.50	1.0071
5000	14000	73.86	56.83	93.20	.6558
7000	14000	55.56	19.21	58.78	.3328
9000	14000	36.16	1.52	36.20	.0419
11000	14000	22.20	-4.80	22.71	-.2130

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
13000	14000	13.54	-6.19	14.89	-.4291
15000	14000	8.48	-5.88	10.32	-.6067
17000	14000	5.56	-5.18	7.60	-.7499
19000	14000	3.84	-4.51	5.92	-.8656
21000	14000	2.79	-3.98	4.86	-.9603
1000	16000	13.30	84.13	85.17	1.4140
3000	16000	33.00	65.71	73.53	1.1054
5000	16000	38.80	40.76	56.27	.8100
7000	16000	34.25	20.22	39.77	.5332
9000	16000	26.01	7.43	27.05	.2784
11000	16000	18.29	.87	18.31	.0476
13000	16000	12.47	-1.98	12.63	-.1578
15000	16000	8.50	-2.99	9.01	-.3378
17000	16000	5.90	-3.18	6.71	-.4942
19000	16000	4.22	-3.07	5.22	-.6293
21000	16000	3.11	-2.88	4.24	-.7463



## MODIFIED GISH AND WAIT MODEL

CHARGE CENTER 1  
HEIGHT COULOMBS  
6096.00 1.52

CHARGE CENTER 2  
HEIGHT COULOMBS  
3048.00 -2.64

CHARGE CENTER 3  
HEIGHT COULOMBS  
.00 .00

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
1000	0	.00	3545.22	3545.22	1.5708
3000	0	-.00	1187.98	1187.98	1.5708
5000	0	.00	250.43	250.43	1.5708
7000	0	.00	-13.31	-13.31	-1.5708
9000	0	-.00	-91.15	91.15	-1.5708
11000	0	.00	-116.55	-116.55	-1.5708
13000	0	.00	-125.58	-125.58	-1.5708
15000	0	.00	-128.98	-128.98	-1.5708
17000	0	.00	-130.27	-130.27	-1.5708
19000	0	.00	-130.73	-130.73	-1.5708
21000	0	.00	-130.86	-130.86	-1.5708
1000	2000	-7475.19	8083.68	11010.19	2.3171
3000	2000	-1617.19	741.00	1778.87	2.7119
5000	2000	-385.38	158.82	416.83	2.7507
7000	2000	-108.76	35.33	114.35	2.8275
9000	2000	-34.87	-3.81	35.07	-3.0328
11000	2000	-12.29	-18.04	21.83	-2.1690
13000	2000	-4.62	-23.54	23.99	-1.7645
15000	2000	-1.78	-25.71	25.77	-1.6399
17000	2000	-.66	-26.54	26.55	-1.5958
19000	2000	-.21	-26.82	26.83	-1.5785
21000	2000	-.02	-26.87	26.87	-1.5716
1000	4000	-7871.57	-10551.76	13164.40	-2.2117
3000	4000	-1323.56	-1069.44	1701.62	-2.4620
5000	4000	-335.01	-203.10	391.77	-2.5966
7000	4000	-108.12	-56.04	121.78	-2.6634
9000	4000	-39.01	-25.36	46.53	-2.5651
11000	4000	-14.98	-18.42	23.74	-2.2536
13000	4000	-5.93	-16.76	17.78	-1.9109
15000	4000	-2.32	-16.30	16.46	-1.7123
17000	4000	-.83	-16.09	16.12	-1.6224
19000	4000	-.21	-15.94	15.94	-1.5838
21000	4000	.05	-15.80	15.80	-1.5677
1000	6000	12714.70	-3423.76	13167.60	-.2630

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
3000	6000	621.85	-834.46	1040.68	-.9304
5000	6000	17.03	-256.63	257.19	-1.5045
7000	6000	-24.18	-89.68	92.89	-1.8342
9000	6000	-15.81	-38.13	41.28	-1.9639
11000	6000	-7.83	-20.88	22.30	-1.9297
13000	6000	-3.46	-14.62	15.02	-1.8032
15000	6000	-1.33	-12.12	12.20	-1.6801
17000	6000	-.35	-11.01	11.02	-1.6024
19000	6000	.08	-10.44	10.45	-1.5631
21000	6000	.25	-10.11	10.12	-1.5462
1000	8000	1201.91	1821.65	2182.42	.9876
3000	8000	580.59	79.03	585.95	.1353
5000	8000	151.69	-83.67	173.24	-.5041
7000	8000	36.74	-56.27	67.20	-.9924
9000	8000	8.45	-31.35	32.47	-1.3076
11000	8000	1.92	-18.54	18.64	-1.4675
13000	8000	.66	-12.49	12.51	-1.5183
15000	8000	.53	-9.61	9.63	-1.5156
17000	8000	.57	-8.19	8.21	-1.5011
19000	8000	.58	-7.43	7.45	-1.4925
21000	8000	.55	-7.00	7.02	-1.4919
1000	10000	147.57	420.51	445.66	1.2333
3000	10000	199.73	141.81	244.95	.6174
5000	10000	107.87	9.84	108.32	.0909
7000	10000	46.23	-17.46	49.42	-.3612
9000	10000	19.05	-16.94	25.49	-.7270
11000	10000	8.28	-12.67	15.13	-.9921
13000	10000	4.05	-9.39	10.23	-1.1635
15000	10000	2.32	-7.38	7.74	-1.2658
17000	10000	1.54	-6.21	6.40	-1.3272
19000	10000	1.14	-5.52	5.64	-1.3677
21000	10000	.89	-5.11	5.18	-1.3982
1000	12000	35.78	144.76	149.11	1.3285
3000	12000	69.59	81.26	106.99	.8626
5000	12000	56.88	26.50	62.75	.4360
7000	12000	34.59	1.73	34.64	.0499
9000	12000	18.92	-5.70	19.76	-.2928
11000	12000	10.21	-6.73	12.23	-.5828
13000	12000	5.75	-6.07	8.36	-.8129
15000	12000	3.47	-5.24	6.28	-.9853
17000	12000	2.27	-4.58	5.11	-1.1104
19000	12000	1.59	-4.12	4.42	-1.2015
21000	12000	1.18	-3.81	3.99	-1.2695
1000	14000	12.26	63.59	64.76	1.3803
3000	14000	28.05	44.40	52.52	1.0074
5000	14000	28.97	22.21	36.51	.6540

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
7000	14000	22.23	7.46	23.45	.3237
9000	14000	14.87	.26	14.87	.0178
11000	14000	9.44	-2.52	9.77	-.2606
13000	14000	5.98	-3.31	6.83	-.5054
15000	14000	3.89	3.36	5.14	-.7120
17000	14000	2.64	-3.19	4.14	-.8794
19000	14000	1.87	-2.99	3.53	-1.0117
21000	14000	1.39	-2.83	3.15	-1.1152
1000	16000	5.18	32.64	33.05	1.4133
3000	16000	12.92	25.55	28.63	1.1025
5000	16000	15.35	15.88	22.08	.8024
7000	16000	13.75	7.81	15.81	.5162
9000	16000	10.66	2.66	10.98	.2445
11000	16000	7.68	-.09	7.68	-.0118
13000	16000	5.38	-1.37	5.55	-.2500
15000	16000	3.78	-1.90	4.22	-.4653
17000	16000	2.69	-2.06	3.39	-.6541
19000	16000	1.97	-2.09	2.87	-.8137
21000	16000	1.48	-2.05	2.53	-.9455

## MODIFIED MALAN MODEL

CHARGE CENTER 1  
HEIGHT COULOMBS  
10000.00 20.10

CHARGE CENTER 2  
HEIGHT COULOMBS  
5000.00 -36.80

CHARGE CENTER 3  
HEIGHT COULOMBS  
2000.00 12.70

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
1000	0	.00	-19578.95	19578.95	-1.5708
3000	0	-.00	3637.27	3637.27	1.5708
5000	0	-.00	3716.05	3716.05	1.5708
7000	0	-.00	1895.86	1895.86	1.5708
9000	0	.00	834.24	834.24	1.5708
11000	0	-.00	318.36	318.36	1.5708
13000	0	-.00	74.38	74.38	1.5708
15000	0	-.00	-41.76	41.76	-1.5708
17000	0	.00	-97.60	-97.60	-1.5708
19000	0	.00	-124.44	-124.44	-1.5708
21000	0	-.00	-137.05	137.05	-1.5708
1000	2000	103233.12	27386.64	106804.06	.2593
3000	2000	-220.84	11093.19	11095.39	1.5907
5000	2000	-2687.20	4168.82	4959.85	2.1434
7000	2000	-1469.13	1723.87	2264.96	2.2766
9000	2000	-694.81	750.14	1022.49	2.3179
11000	2000	-323.74	326.33	459.67	2.3522
13000	2000	-152.78	131.82	201.79	2.4297
15000	2000	-73.20	39.62	83.24	2.6456
17000	2000	-35.29	-4.78	35.62	-3.0069
19000	2000	-16.79	-26.07	31.01	-2.1430
21000	2000	-7.61	-35.92	36.72	-1.7795
1000	4000	-106052.11	132575.97	169774.69	2.2455
3000	4000	-22433.32	12080.09	25479.06	2.6476
5000	4000	-6881.11	2186.94	7220.28	2.8339
7000	4000	-2651.36	604.03	2719.30	2.9176
9000	4000	-1133.96	219.37	1154.98	2.9505
11000	4000	-514.44	83.59	521.19	2.9805
13000	4000	-242.18	22.49	243.22	3.0490
15000	4000	-116.43	-7.75	116.69	-3.0752
17000	4000	-56.17	-22.75	60.60	-2.7568
19000	4000	-26.49	-29.72	39.81	-2.2988
21000	4000	-11.62	-32.40	34.42	-1.9151
1000	6000	-112747.42	-120485.62	165011.41	-2.3230
3000	6000	-24307.53	-12275.94	27231.50	-2.6739
5000	6000	-6793.48	-3155.99	7490.77	-2.7067

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<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
7000	6000	-2468.76	-1113.56	2708.28	-2.7179
9000	6000	-1045.96	-455.83	1140.97	-2.7306
11000	6000	-481.72	-211.68	526.18	-2.7276
13000	6000	-231.42	-113.86	257.91	-2.6844
15000	6000	-112.84	-72.00	133.85	-2.5737
17000	6000	-54.37	-52.51	75.59	-2.3736
19000	6000	-24.85	-42.35	49.10	-2.1015
21000	6000	-9.82	-36.26	37.57	-1.8354
1000	8000	6212.02	-60396.07	60714.70	-1.4683
3000	8000	-268.39	-18165.98	18167.96	-1.5856
5000	8000	-1294.72	-5611.02	5758.46	-1.7976
7000	8000	-840.03	-2057.74	2222.60	-1.9584
9000	8000	-468.22	-867.77	986.04	-2.0656
11000	8000	-251.55	-408.78	479.98	-2.1225
13000	8000	-131.84	-212.91	250.42	-2.1252
15000	8000	-66.51	-122.65	319.53	-2.0677
17000	8000	-31.09	-78.18	84.14	-1.9493
19000	8000	-12.11	-54.75	56.07	-1.7885
21000	8000	-2.16	-41.49	41.54	-1.6227
1000	10000	178385.59	-10509.62	178694.91	-.0588
3000	10000	15647.43	-6656.05	17004.27	-.4022
5000	10000	3283.88	-3374.60	4708.69	-.7990
7000	10000	794.46	-1638.40	1802.85	-1.1193
9000	10000	188.63	-817.80	839.28	-1.3441
11000	10000	36.77	-428.97	430.54	-1.4853
13000	10000	3.73	-238.40	238.43	-1.5552
15000	10000	1.04	-141.21	141.22	-1.5634
17000	10000	4.47	-89.55	89.66	-1.5209
19000	10000	7.70	-60.86	61.35	-1.4449
21000	10000	9.56	-44.18	45.20	-1.3577
1000	12000	15343.80	27077.87	31123.04	1.0553
3000	12000	9643.77	3652.56	10312.30	.3621
5000	12000	3639.16	-331.55	3654.24	-.0909
7000	12000	1391.71	-660.15	1540.34	-.4429
9000	12000	562.67	-487.56	744.52	-.7140
11000	12000	244.91	-314.33	398.48	-.9089
13000	12000	117.72	-198.32	230.62	-1.0351
15000	12000	64.31	-127.23	142.56	-1.1028
17000	12000	40.48	-84.50	93.70	-1.1241
19000	12000	28.86	-58.65	65.36	-1.1135
21000	12000	22.48	-42.67	48.23	-1.0860
1000	14000	2204.57	7239.22	7567.46	1.2752
3000	14000	3376.22	3169.39	4630.75	.7538
5000	14000	2220.67	792.93	2357.99	.3430
7000	14000	1197.03	16.01	1197.14	.0134
9000	14000	620.33	-159.92	640.61	-.2523
11000	14000	327.34	-162.03	365.25	-.4596
13000	14000	181.07	-127.25	221.31	-.6126

<u>X</u>	<u>Y</u>	<u>X FIELD</u>	<u>Y FIELD</u>	<u>TOTAL</u>	<u>THETA</u>
15000	14000	106.90	-93.18	141.81	-.7170
17000	14000	67.96	-67.41	95.72	-.7814
19000	14000	46.49	-49.41	67.85	-.8158
21000	14000	33.92	-37.15	50.31	-.8308
1000	16000	604.37	2822.63	2886.61	1.3599
3000	16000	1259.94	1806.48	2202.46	.9618
5000	16000	1160.41	819.86	1420.81	.6151
7000	16000	815.26	271.42	859.25	.3214
9000	16000	516.19	39.06	517.67	.0755
11000	16000	317.21	-40.16	319.74	-.1259
13000	16000	196.48	-57.73	204.79	-.2858
15000	16000	125.36	-54.10	136.53	-.4074
17000	16000	83.34	-45.11	94.76	-.4961
19000	16000	57.98	-36.24	68.37	-.5586
21000	16000	42.17	-28.97	51.16	-.6010

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## APPENDIX B

The following program, written for the HP2100 computer, is used to minimize the percentage of variation as shown in Equation 8 by varying the values of three charge centers A, B and C which are located at YA, YB and YC. One charge center is modified at a time beginning with A, then B, then C, back to A and so on until an oscillation is noticed in the percentage of variation. At that point, the value of INC can be decreased and the program restarted using the A, B and C values which were computed as the oscillation began. The output gives the values of A, B and C in coulombs and the corresponding sum of the squares of the percentage of variations.

The initial value of INC was 0.1. Oscillation using this value of INC occurred at A = 6.8, B = 13.8 and C = 3.4 for the Simpson and Robinson model. ESQRT equalled  $.187 \times 10^4$ . Oscillation in the Gish and Wait model occurred at A = 2.7 and B = 5 with ESQRT equalling  $.813 \times 10^3$  while in the Malan model it occurred at A = 22.9, B = -40, C = 10 and ESQRT =  $.269 \times 10^4$ .

Changing INC to equal .01 and using the A, B and C values found in the previous step resulted in the following values for the modified models:

<u>MODIFIED MODEL</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>ESQRT</u>
Simpson & Robinson	4.6	-11.3	5.97	522
Gish & Wait	1.52	- 2.64	0.0	624
Malan	20.1	-36.8	12.7	799



```

PROGRAM FIELD
REAL INC,MULT
DIMENSION EFLD(9),X(9),EF(9),Y(9)
11 READ (5,3) A,B,C,YA,YB,YC
3  FORMAT (6F10.1)
   IF (A) 13,12,13
13 READ (5,4) A1,A2,A3,A4,A5,A6,A7
4  FORMAT (7A4)
   WRITE (6,5) A1,A2,A3,A4,A5,A6,A7
5  FORMAT (1H1,45X,7A4,/,3X,"CHARGE CENTER A",5X,"CHARGE CENTER B", 5
1X,"CHARGE CENTER C",/,2X,"HEIGHT",3X,"COULOMBS",3X,"HEIGHT",3X,"CO
2ULOMBS",3X,"HEIGHT",3X,"COULOMBS")
   WRITE (6,3) YA,A,YB,B,YC,C
   VALUE=A
   INC=.1
   J=1
   EF(1)=-1310.
   EF(2)=-1160.
   EF(3)=-760.
   EF(4)=-460.
   EF(5)=-260.
   EF(6)=-135.
   EF(7)=-60.
   EF(8)=1.
   EF(9)=33000.
   X(1)=2500.
   X(2)=5000.
   X(3)=7500.
   X(4)=10000.
   X(5)=12500.
   X(6)=15000.
   X(7)=17500.
   X(8)=20000.
   X(9)=13100.
   MULT=1./(4.*3.1415*8.854*(10.**(-12)))
131 II=1
    K=1
130 EXQRT=0.
    DO 10 I=1,8,1
      EFLD(I)=MULT*((2.*YA*A/(X(I)**2.+YA**2.))**1.5)+(2.*YB*B/(X(I)**2.+
1YB**2.))**1.5)+(2.*YC*C/(X(I)**2.+YC**2.))**1.5))
      ESQRT=ESQRT+(EF(I)-EFLD(I))*(EF(I)-EFLD(I))/(EF(I)*EF(I))
10  CONTINUE
      EFLD(9)=MULT*(A/(Y(9)-YA)**2.-A/(Y(9)+YA)**2.+B/(Y(9)-YB)**2.-B/(Y
1(9)+YB)**2.+C/(Y(9)-YC)**2.-C/(Y(9)+YC)**2.))

```

```

ESQRT=ESQRT+(EF(9)-EFLD(9))*(EF(9)-EFLD(9))/(EF(9)*EF(9))
ESQRT=ESQRT*100.
IF (II-1) 20,20,30
20  ESMIN=ESQRT
    GO TO 40
30  IF (ESQRT-ESMIN) 20,20,50
50  IF (II-2) 60,60,70
60  K=-K
    VALUE=VALUE+K*INC
40  II=II+1
    VALUE=VALUE+K*INC
    IF (ESMIN-100.) 180,180,80
80  IF (J-1) 90,90,100
90  A=VALUE
    GO TO 130
100 IF (J-2) 110,110,120
110 B=VALUE
    GO TO 130
120 C=VALUE
    GO TO 130
70  IF (J-1) 140,140,150
140 A=VALUE
    VALUE=B
    J=2
    GO TO 190
150 IF (J-2) 160,160,170
160 B=VALUE
    VALUE=C
    J=3
    GO TO 190
170 C=VALUE
    VALUE=A
    J=1
190 WRITE (6,2) A,B,C,ESMIN
2    FORMAT (4E10.3)
    GO TO 131
180 WRITE (6,6)
6    FORMAT(//,1X,"CHARGE A",2X,"CHARGE B",2X,"CHARGE C",3X,"EFLD(1)",3
1X,"EFLD(2)",3X,"EFLD(3)",3X,"EFLD(4)",3X,"EFLD(5)",3X,"EFLD(6)",3X
2,"EFLD(7)",3X,"EFLD(8)",3X,"EFLD(9)")
    WRITE (6,1) A,B,C,EFLD(1),EFLD(2),EFLD(3),EFLD(4),EFLD(5),EFLD(6),
1EFLD(7),EFLD(8),EFLD(9)
1    FORMAT (12E10.3)
    GO TO 11
12  END

```

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Section IV

Aircraft Noise Suppression -  
Electronic Acoustic Noise Suppression

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## INTRODUCTION

An alternative to passive sound insulation is active electronic control. In the approach, described earlier by Olson [1,2] and others [3,4], one connects a microphone, an inverting amplifier, and a loudspeaker in a negative feedback fashion to reduce noise. In this system (figure 1), the electric signal produced by the microphone from an incident sound pressure  $P_i$ , is amplified by an inverting amplifier with a gain of  $-K_a$ . The amplifier drives the loudspeaker, producing an output pressure  $P_o$ . This pressure is summed acoustically with the incident sound pressure  $P_i$ . The resulting pressure at the microphone is the sound pressure error signal,  $P_e$ ;

$$P_e = P_i + P_o$$

The resulting sound pressure level  $P_e$  at the microphone has been shown to be less than the incident sound pressure level  $P_i$ , [1,2,5]. If the microphone is placed near one's ear, a considerable reduction in the noise level is observed. However, airborne noise reduction through active electronic control is confined to a fairly narrow range of frequencies.

This treatise will show that signal time delays from wave propagation through the acoustic medium and inherent transducer delay are responsible for this narrow bandwidth limitation. A classical method of lag-lead compensation will be used to improve the bandwidth of airborne-noise-

reduction systems developed in the references [1,2,5]. Finally, lag-lead compensation using maximally-flat (Butterworth) and equal-ripple (Chebyshev) polynomials will be developed to further extend the bandwidth.

### MATH MODEL

The active compensation system of figure 1 is depicted schematically by the block diagram in figure 2. Here,  $-K_a$  represents the gain of an ideal amplifier.  $G_m(s)$ ,  $G_f(s)$ , and  $G_s(s)$  are the transfer functions of the microphone, filter, and loudspeaker respectively, in the  $s$  domain.

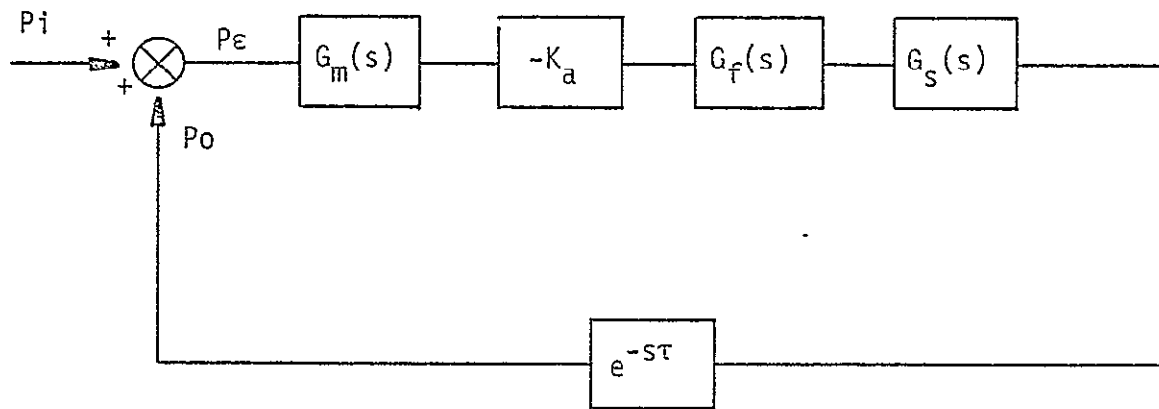
(The transfer function  $G_f(s)$  is the principle topic of this paper and is discussed below in detail.) System time delay  $\tau$  is represented by  $e^{-s\tau}$ . This transformation from the time domain to the frequency domain has been accomplished according to the Laplace shift theorem:

$$L\{y(t-\tau) U_\tau(t)\} = e^{-s\tau} Y(s), \quad \text{where } U_\tau(t) = \begin{cases} 0 & t < \tau \\ 1 & t \geq \tau \end{cases} \quad (2)$$

$$\tau = \tau_p + \tau_t, \quad \text{where } \tau_p = \text{wave propagation delay,} \quad (3) \\ \text{and } \tau_t = \text{total transducer delay.}$$

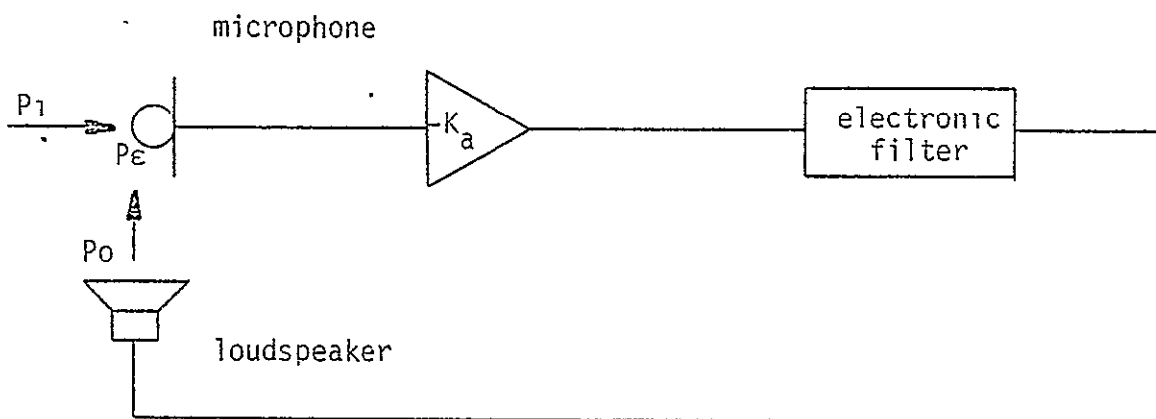
Time delay  $\tau$  is the sum of transducer delay and acoustic wave propagation delay. When time delay is present in a feedback control system, numerical analysis of that system is difficult because of the introduction of the transcendental function  $e^{-s\tau}$ , in the transfer function. However, a graphical approach is often adequate for stability analysis of feedback systems with time delay. This is the method used here.





Block diagram representation

Figure 2



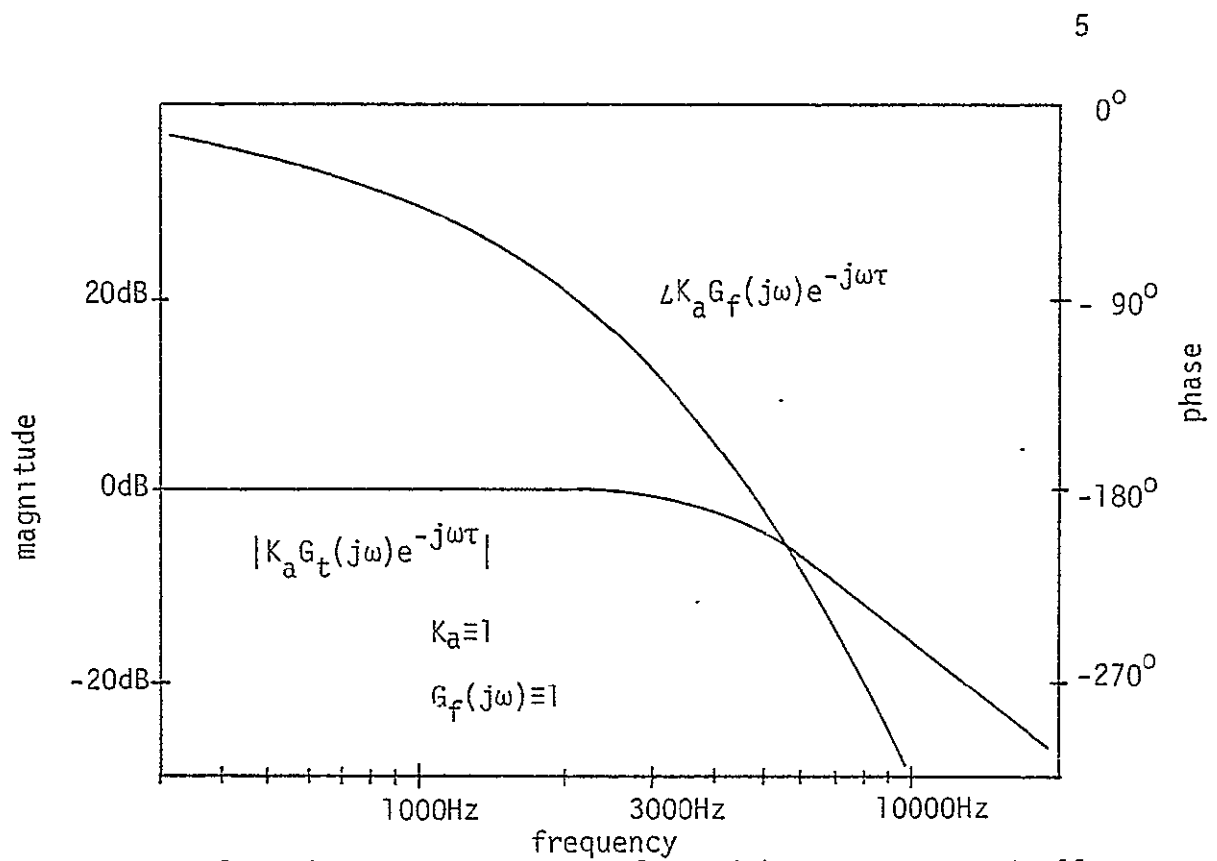
Active compensation scheme

Figure 1

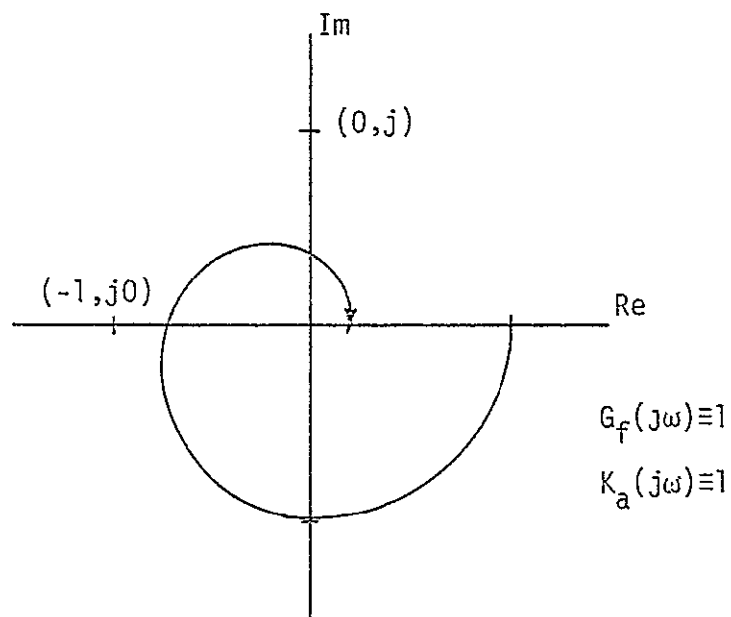
The open-loop response (where  $G_f(s) \equiv 1$ ) of an active electronic airborne-noise controller are shown in figures 3a and 3b. At the higher frequencies, the phase-frequency relationship becomes linear, indicating the presence of time delay. This is graphically illustrated in figure 4. Note that the system phase is the sum of the phase associated with two poles and the phase that represents pure time delay. At high frequencies, for which all phase associated with poles and zeros can be considered a constant, the delay  $\tau$  of equation (4) is simply the slope of the system phase function.

$$\angle e^{-j\omega\tau} = -\omega\tau \quad (4)$$

For the system represented in figures 3a, 3b, and 4 a time delay of 49  $\mu$ secs is determined. Meeker [5] achieved this result with a microphone-speaker separation of 3.1 mm (.125 inch), which accounts only for 9  $\mu$ secs of signal delay due to acoustic wave propagation (with an acoustic wave velocity of 347 meters/sec.) Approximately 40  $\mu$ secs of delay must be attributed to the transducers. This conclusion is substantiated by the work of C.A. Ewaskio and O.K. Mawardi [6]. Their measurements of phase shift in loudspeakers indicated time delays of the magnitude found above. Transducer delay  $\tau_t$  in equation (3) is equal to 40  $\mu$ secs for the system of figures 3a and 3b.

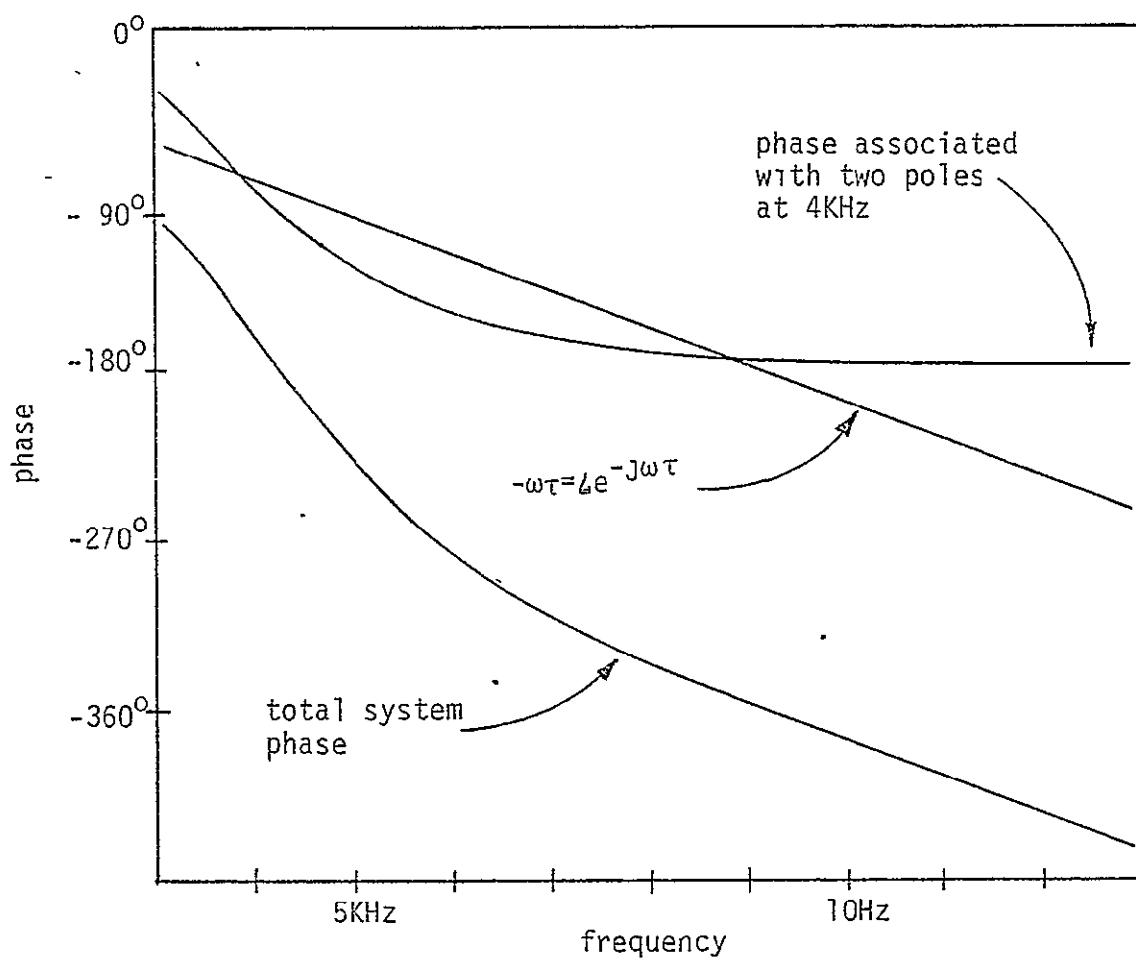


Open-loop frequency response of an airborne-noise controller  
Figure 3a



nyquist diagram of figure 3a

Figure 3b



Open-loop phase with  $G_f(j\omega) \equiv 1$

Figure 4

The magnitude function in figure 3a indicates a bandwidth of 4 KHz, at which point it rolls-off at -12 dB/octave. For this system to achieve stability without compensation, the open-loop gain must be made less than unity.

Lowering the gain of the system, however, decreases its effectiveness. The acoustic noise intensity  $I_r$  is calculated by comparing the sound pressure error signal  $P_e$  with the original incident sound pressure  $P_i$ .  $I_r$  in decibels is:

$$I_r(j\omega) = 20 \log_{10} \frac{P_i(j\omega)}{P_e(j\omega)} \quad (5)$$

The sound pressure is measured at the microphone's diaphragm,  $P_e$  is measured with the active electronic control functioning,  $P_i$  is measured with the system off.

From the block diagram (figure 2),  $P_e$  for real frequencies is,

$$P_e(j\omega) = \frac{P_i(j\omega)}{1 + K_a G_t(j\omega) e^{-j\omega\tau}}, \quad (6)$$

$$\text{where } G_t(j\omega) \triangleq G_m(j\omega) G_f(j\omega) G_s(j\omega).$$

Substituting into equation (5),  $I_r$  becomes

$$I_r(j\omega) = 20 \log_{10} |1 + K_a G_t(j\omega) e^{-j\omega\tau}|. \quad (7)$$

For those real frequencies  $\omega$ , where the open-loop gain is much greater than unity,  $|K_a G_t(j\omega) e^{-j\omega\tau}| \gg 1$ , the reduction in the acoustic noise intensity at the microphone's diaphragm is approximately equal to the system's open-loop gain.

$$I_r(j\omega) \approx 20 \log_{10} |K_a G_t(j\omega)|, \text{ since } |e^{-j\omega\tau}| \equiv 1. \quad (8)$$

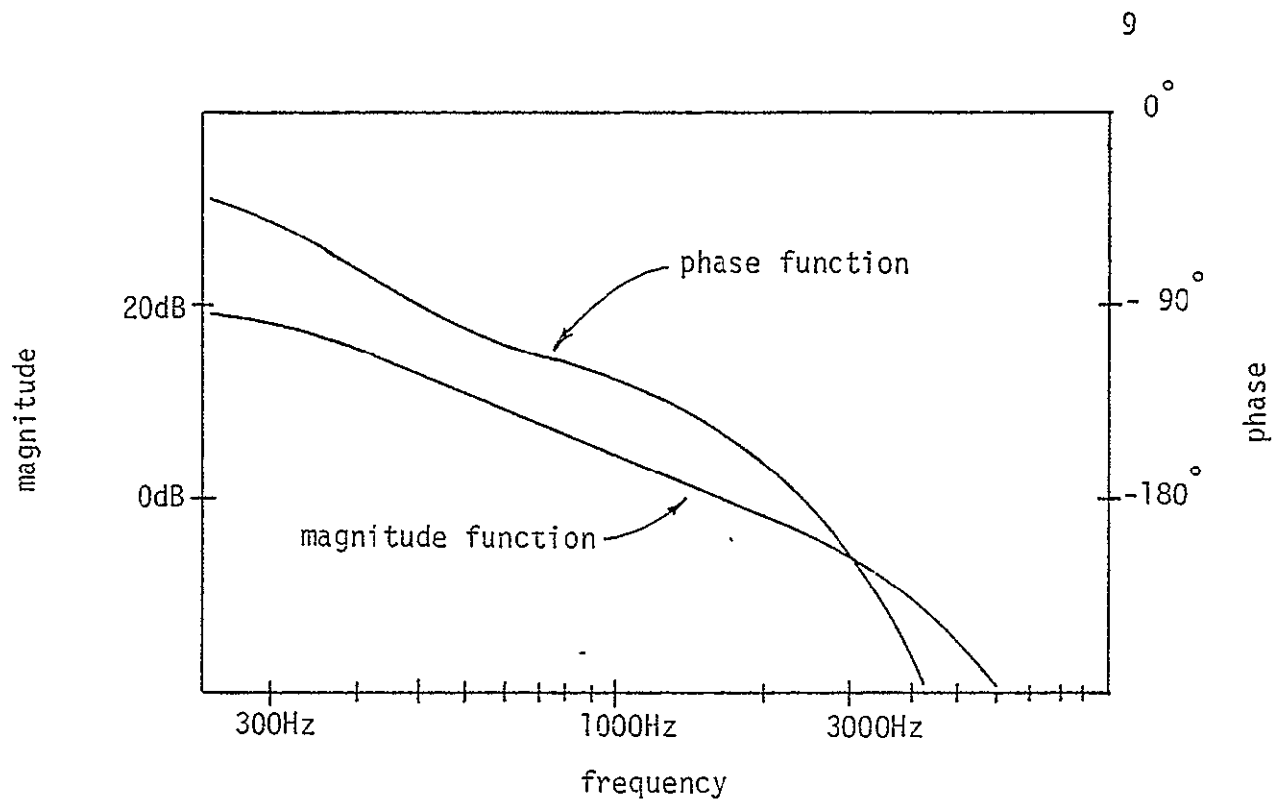
Thus, for maximum noise reduction, the open-loop gain should be kept as high as possible.

### SYSTEM DESIGN

To obtain both stability and high gain, bandwidth of the system must be sacrificed. This conclusion will be made clear through graphical stability analysis in the form of a Nyquist diagram. A polar plot of the open-loop frequency response of  $K_a G_t(j\omega) e^{-j\omega\tau}$  for  $-\infty < \omega < \infty$  is constructed. This plot is referred to as a Nyquist diagram of  $K_a G_t(j\omega) e^{-j\omega\tau}$ , see figure 3b. A minimal-phase (no right half-plane zeros) closed-loop system is stable if and only if the Nyquist diagram of  $K_a G_t(j\omega) e^{-j\omega\tau}$  does not encircle the  $(-1, j0)$  critical point on the complex plane.

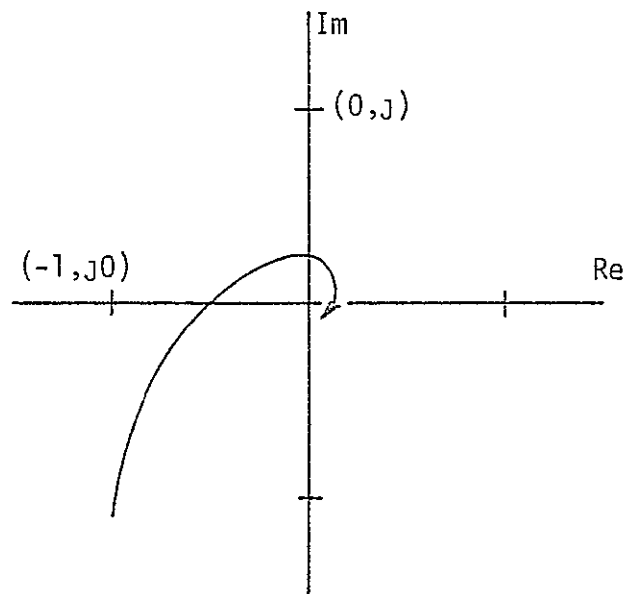
Olson [1,2] and Meeker [5] used lag compensation to obtain stability, simultaneously obtaining high open-loop gain over a narrow range of frequencies, see figures 5a and 5b. To obtain a gain of 20dB, the bandwidth had to be lowered to 200Hz. The compensated noise-reducing system of figure 4a has a filter  $G_f(j\omega)$ , where

$$G_f(j\omega) = \frac{1}{j\omega/1250 + 1}. \quad (9)$$



System developed by Olson [1,2]

Figure 5a



Nyquist diagram of Figure 5a

Figure 5b

The filter  $G_f(j\omega)$  has one pole at 200 Hz. Thus, the Nyquist diagram avoids encirclement of the  $(-1, j0)$  critical point, producing a stable system.

From equation (8) and figure 5a, reduction in the noise intensity is found to be approximately 20dB up to a frequency of 200 Hz. To increase the bandwidth of the system in figures 4a and 4b one would like to raise the crossover frequency  $\omega_{-180}$  of  $\angle e^{-j\omega\tau}$ . ( $\omega_{-180}$  is defined to be the frequency where  $\angle e^{-j\omega\tau}$  is equal to -180 degrees). One would like to move  $\omega_{-180}$  to infinity, that is eliminate the time delay; however, this is not physically realizable. In the airborne-noise-reduction system investigated, a large percentage of the time delay is associated with transducers. With present state-of-the-art transducers, little can be done to reduce the inherent time delay. Therefore, to avoid encirclement of the  $(-1, j0)$  point on the Nyquist diagram, the magnitude function must be less than unity before the frequency  $\omega_{-180}$ . Consequently, this frequency,  $\omega_{-180}$  must be considered the upper limit on an airborne-noise-reduction system's bandwidth.

If the bandwidth is to be increased, it must be done through manipulation of the magnitude function. An increase in the bandwidth may be realized if the magnitude function is made to roll-off at a greater rate than that of the one-pole filter in figure 5a. This allows the -3dB bandwidth to move closer to the upper limit,  $\omega_{-180}$ . In a minimal-phase system increased roll-off is always accompanied by additional phase. Hence, trade-offs must be made.



Lag-lead compensation is commonly used to reduce gain while minimizing excess phase. The transfer function of a lag-lead compensator consists of a simple pole-zero pair:

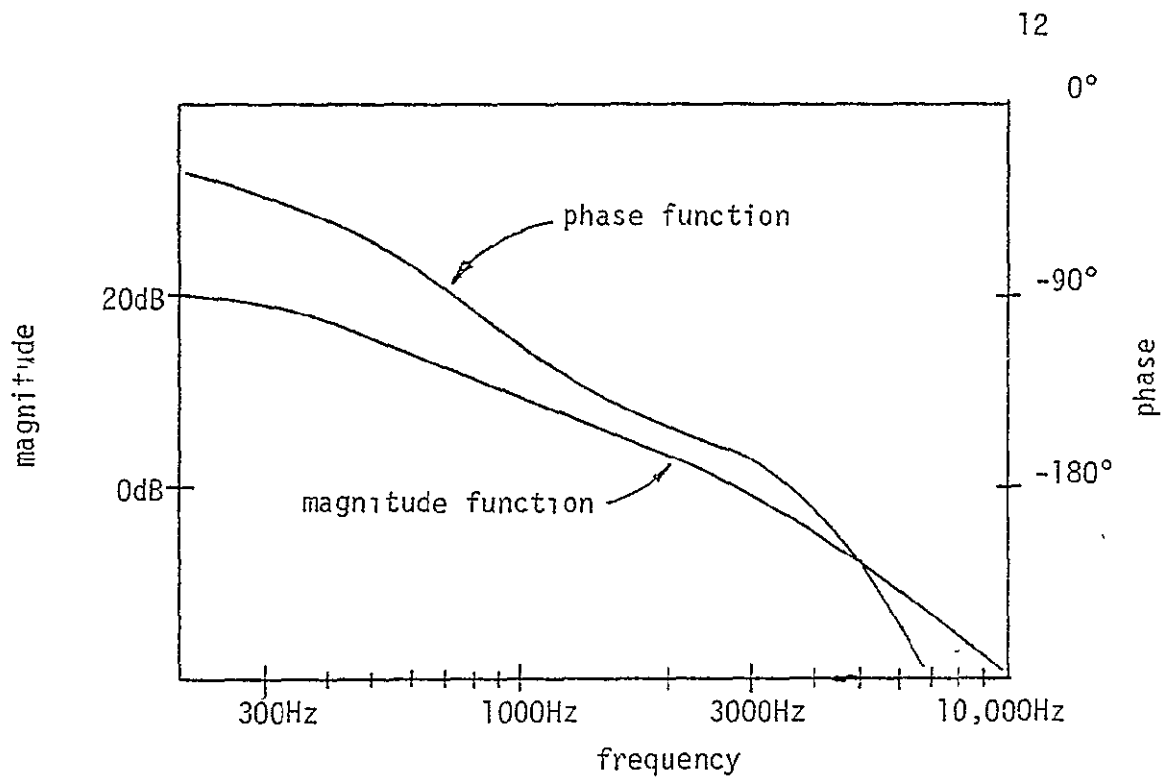
$$G_f(j\omega) = \frac{1+j\omega/\omega_z}{1+j\omega/\omega_p} \quad (10)$$

The pole is always nearer the origin of the  $s$  plane than the zero. Note that the magnitude function,  $G_f^1(j\omega)$ , has an initial gain of unity. As  $\omega$  approaches infinity, the gain becomes  $\omega_p/\omega_z$ . Moreover, the phase function returns to zero as  $\omega$  goes to infinity. Lag-lead compensation allows the magnitude function to decrease while introducing smaller amounts of excess phase than lag compensation.

The use of a first-order (both denominator and numerator are first order polynomials) lag-lead compensator in the airborne-noise-reduction systems of the references [1,2,5] would extend the bandwidth to 300Hz, see figures 6a and 6b. The pole-zero location of the compensator have been determined graphically and found to be

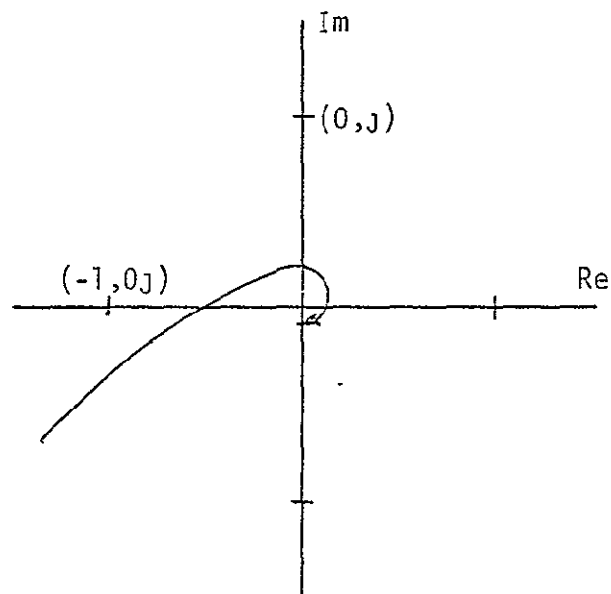
$$G_f^1(j\omega) = \frac{1+j\omega/2.51 \times 10^4}{1+j\omega/1880} \quad (11)$$

A graphical approach for finding the pole and zero locations of a lag-lead compensator is quite adequate for a single-input, single-output closed-loop system. The location of the pole and zero is left to the designer. The



Olson's system with first-order lag-lead compensation

Figure 6a



Nyquist diagram of Figure 6a

Figure 6b

proper phase and gain margins may then be selected for desired overshoot and damping. Insofar as the noise-reduction system can be approximated by a second-order system, the phase-gain margin concept will be a graphical convenience which can provide approximate closed-loop analysis by inspection. Criteria for phase and gain margins will not be discussed, except for the statement that phase margin greater than 55 degrees and a gain margin greater than 6dB are desirable. This gives an approximate closed-loop damping  $>.55$ .

The ideal normalized (the -3dB bandwidth frequency, defined as  $\omega_{-3dB}$ , is equal to one) lag-lead compensator is characterized by a magnitude that is unity for frequencies,  $\omega \leq 1$  and can be represented by  $1/K_f$ , for frequencies  $\omega > 1$ . These ideal properties are not attainable with constant, lumped, and linear networks, therefore the requirements imposed by the ideal characteristics can be only approximated. This is done by allowing the magnitude and phase to stay within prescribed limits of the ideal, see figures 7a and 7b.

It is evident from the magnitude function graphed in figures 8a and 8b that cascading more than one first-order lag-lead compensator results in a magnitude function  $|G_f'(j\omega)|$  that tends away from the ideal. Also from figure 8b, it is evident that excess phase is increased with increased number of cascaded filters.

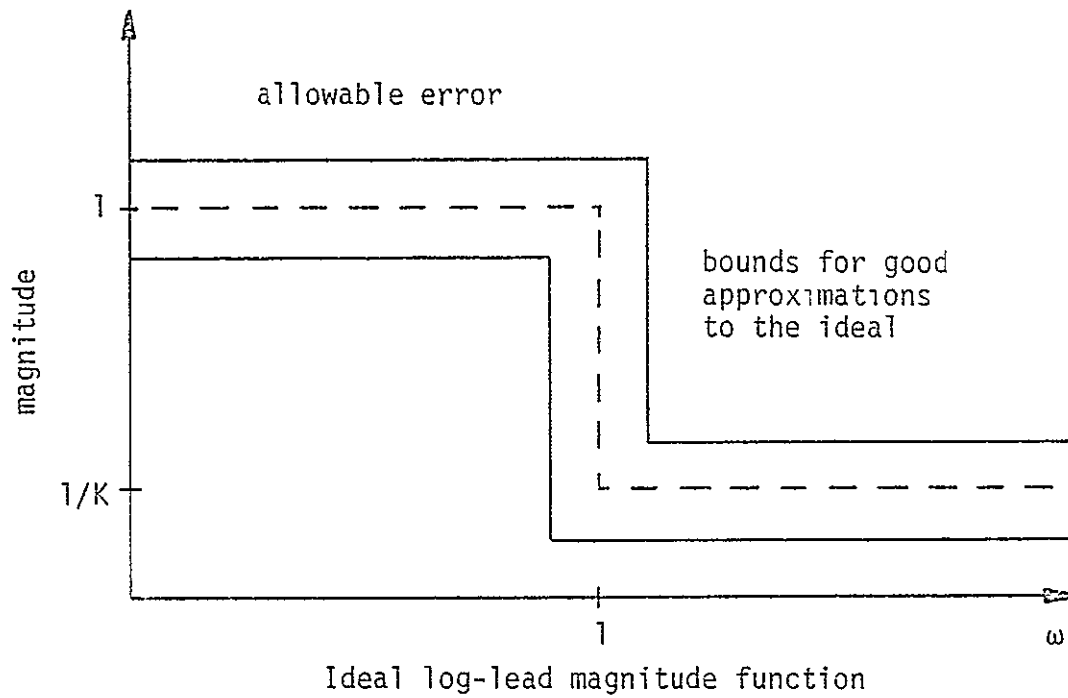


Figure 7a

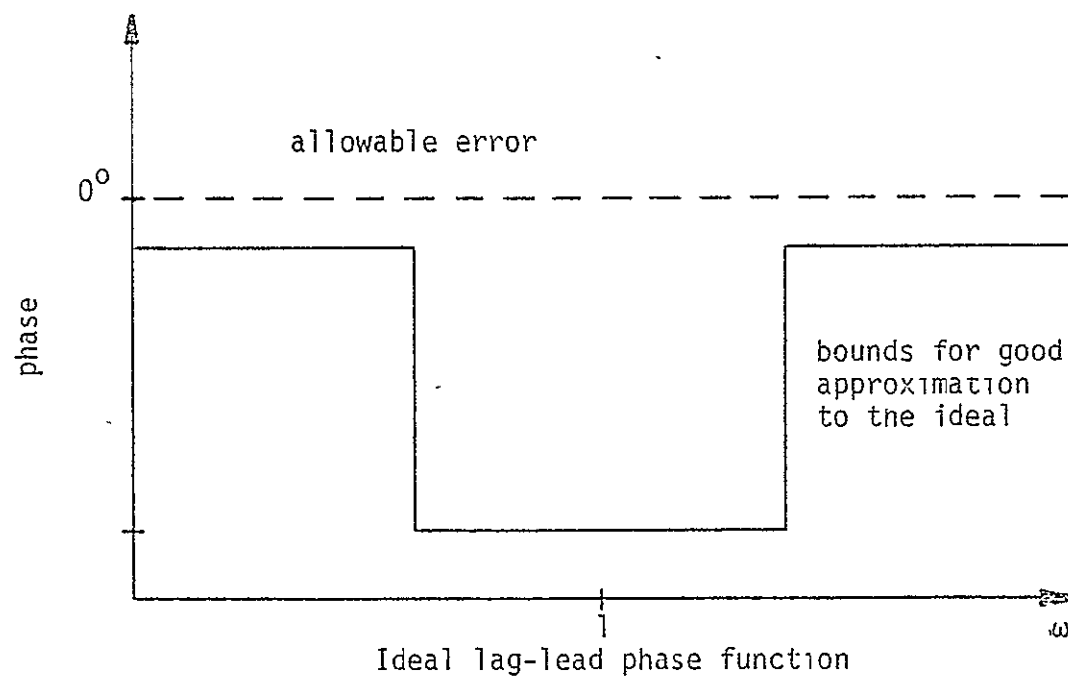


Figure 7b

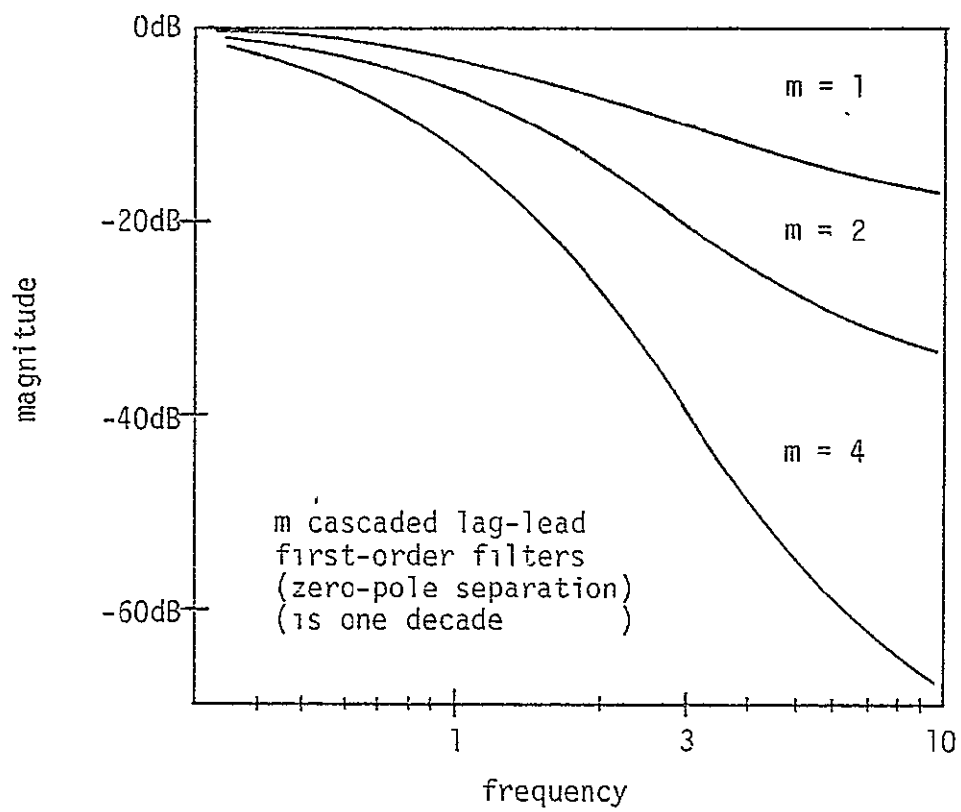


Figure 8a

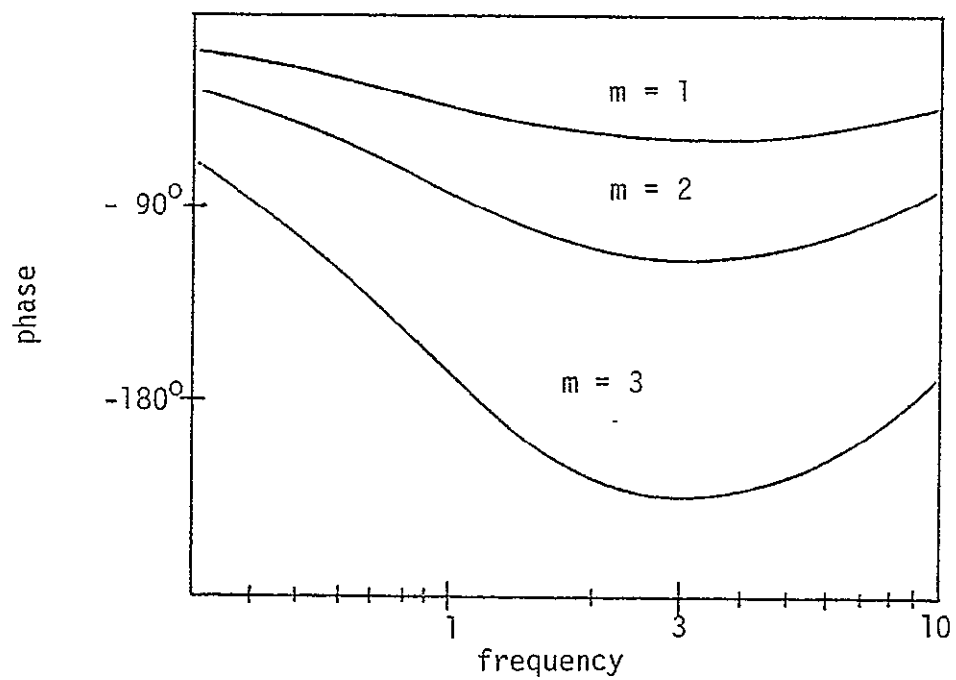


Figure 8b

### BUTTERWORTH-CHEBYSHEV LAG-LEAD COMPENSATION

Functions of the form

$$G_f(s) = \frac{N(s)}{D(s)}, \quad (12)$$

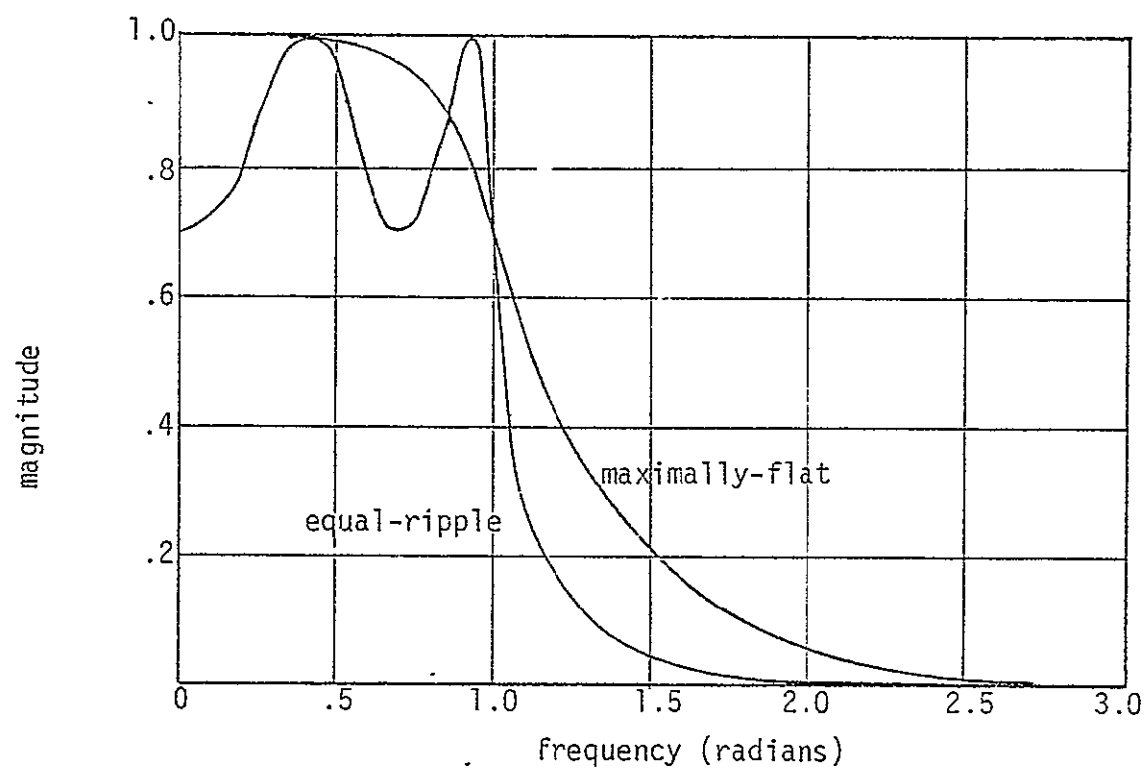
which obey the specified constraints of figures 7a and 7b, represent approximations to the ideal lag-lead filter. Combinations of the maximally-flat (Butterworth) and the equal-ripple (Chebyshev) polynomials provide candidate approximations to the ideal.

The excess phase associated with the maximally-flat function increases with frequency at a much smaller rate than that of the equal-ripple polynomial, see figure 9a and 9b. Of the four ratio combinations, representing the equal-ripple polynomial by  $N(s)$  (which contributes positive phase) and the maximally-flat polynomial by  $D(s)$  (which contributes relative little negative phase) yields the most desirable phase function.

A suitable representation of this lag-lead compensator can be obtained by starting with the general expression of the magnitude-squared function. Let

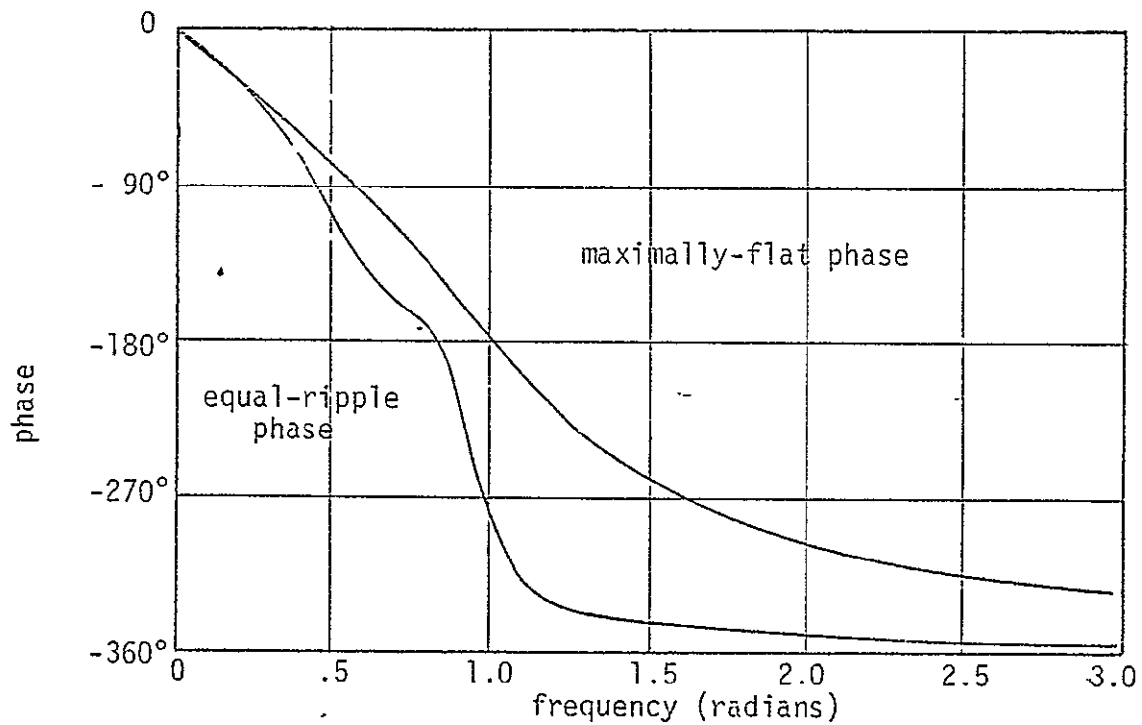
$$|G_f(j\omega)|^2 = \frac{N(\omega^2)}{D(\omega^2)}, \quad (13)$$

for real frequencies  $\omega$ .



Fourth-order Butterworth and Chebyshev magnitude function

Figure 9a



Fourth-order Butterworth and Chebyshev phase function

Figure 9b

For frequencies  $\omega \leq 1$  the magnitude-squared function of the normalized filter  $G_f(j\omega)$  should be approximately unity. For frequencies  $\omega > 1$  the function should not depart appreciably from  $1/K_f^2$ . The filter design to be discussed in detail is

$$|G_f''(j\omega)|^2 = \frac{1 + \epsilon^2 C_n^2\left(\frac{1}{\alpha\omega}\right)}{K_f^2 + \epsilon^2 \frac{2^{2(n-1)}}{(\alpha\omega)^{2n}}}, \quad (14)$$

where the  $C_n(x)$  are called the Chebyshev polynomials.

$$C_n^2\left(\frac{1}{\alpha\omega}\right) = \cos^2\left(n \cos^{-1} \frac{1}{\alpha\omega}\right) \quad (15a)$$

$$= \cosh^2\left(n \cosh^{-1} \frac{1}{\alpha\omega}\right). \quad (15b)$$

Although either form of  $C_n^2(1/\alpha\omega)$  can be employed, it is convenient to use equation (15a) for frequencies  $1/\alpha\omega \leq 1$  and equation (15b) for  $1/\alpha\omega > 1$ .

It will presently be shown that  $|G_f''(j\omega)|^2$  is a ratio of an equal-ripple polynomial ( $N(\omega^2)$ ) to a maximally-flat polynomial ( $D(\omega^2)$ )

For real frequencies  $\omega$ , when  $\omega \rightarrow 0$ ,

$$|G_f''(j\omega)|^2 \underset{\omega \rightarrow 0}{\approx} \frac{\epsilon^2 C_n^2\left(\frac{1}{\alpha\omega}\right)}{\epsilon^2 \frac{2^{2(n-1)}}{(\alpha\omega)^{2n}}} = 1 \quad (16a)$$

because

$$\cosh^2\left(n \cosh^{-1} \frac{1}{\alpha\omega}\right) \approx 2^{2(n-1)} \left(\frac{1}{\alpha\omega}\right)^{2n}, \text{ for } \frac{1}{\alpha\omega} > 1 \quad (16b)$$



For real frequencies  $\omega$ , when  $\omega \rightarrow \infty$ ,

$$\left| G_f''(j\omega) \right|^2 \underset{\omega \rightarrow \infty}{\cong} \begin{cases} 1/K^2 & \text{for } n \text{ odd} \\ (1+\epsilon^2)/K^2 & \text{for } n \text{ even,} \end{cases} \quad (17a)$$

when  $n$  is the order of the polynomial.

This is true because

$$\cos^2(n \cos^{-1} \frac{1}{\alpha\omega}) = \cos^2(n \frac{\pi}{2}), \text{ for } \frac{1}{\alpha\omega} \ll 1. \quad (17b)$$

For frequencies  $\omega > 1/\alpha$ ,  $C_n^2(1/\alpha\omega)$  is a cosine-squared function. Therefore

$$0 \leq |C_n^2(1/\alpha\omega)| \leq 1. \quad (18)$$

The magnitude characteristic of  $N(\omega^2)$  is equal-ripple in nature, the numerator of  $\left| G_f''(j\omega) \right|^2$  will swing between 1 and  $1+\epsilon^2$  exactly  $n$  times, for  $\omega > 1/\alpha$ .

Note that

$$|N(\omega^2)| = 1 \quad \text{when } C_n^2(1/\alpha\omega) = 0.$$

This occurs when  $\omega = \frac{1}{\alpha} \sec \frac{\pi k}{2n}$  (where  $k = 1, 3, 5, \dots$ ).

Also,

$$|N(\omega^2)| = 1+\epsilon^2 \quad \text{when } C_n^2(\frac{1}{\alpha\omega}) = 1.$$

This occurs when  $\omega = \frac{1}{\alpha} \sec \frac{\pi k}{n}$  (where  $k=0, 1, 2, \dots$ ).

Because the parameter  $\epsilon$  controls the amount of ripple for  $\omega > 1/\alpha$ , it is called the ripple factor.

All filter designs discussed will be normalized about the filter's -3dB frequency,  $\omega_{-3dB}$ ; thus  $\omega_{-3dB} = 1$ . The normalizing parameter  $\alpha$  is the value for which

$$|G_f''(j\omega_{-3dB})|^2 = \frac{1}{2} \quad (19)$$

The value of  $\alpha$  must be solved by a successive-approximation. The value of  $\alpha$  as a function of  $K_f/\epsilon$  for different  $n$  is shown in figure 10. If the approximation (16b) is valid for  $\omega=1$ , then

$$C_n^2\left(\frac{1}{\alpha}\right) \cong 2^{2(n-1)} \frac{1}{(\alpha)^{2n}}, \text{ for } \frac{1}{\alpha} > 1.$$

Substituting for  $C_n^2(1/\alpha)$  in equation (14),

$$|G_f''(j\omega)|_{\omega=1}^2 = \frac{\epsilon^2 2^{2(n-1)} \frac{1}{\alpha^{2n}}}{K_f^2 + \epsilon^2 \frac{2^{2(n-1)}}{\alpha^{2n}}} = \frac{1}{2}.$$

Solving for the normalizing parameter  $\alpha$ ,

$$\alpha = \left(\frac{\epsilon}{K_f}\right)^{1/n} 2^{(n-1)/n} < 1. \quad (20)$$

This result is displayed in figure 10, where it is compared with the calculated value of the normalizing parameter  $\alpha$ . Note that equation (20) is valid for  $\alpha$  less than unity, because  $\cosh(x)$  in the Chebyshev polynomial may be approximated by  $e^x/2$ . Therefore, equation (20) can be considered valid when

$$\frac{K_f}{\epsilon} > 2^{n-1}. \quad (21)$$

Also of interest is the frequency  $\omega = 1/\alpha$ .  $|G_f''(j\omega)|^2$  becomes

$$\left| G_f''(j\omega) \right|_{\omega=\frac{1}{\alpha}}^2 \approx \frac{1+\epsilon^2}{K_f^2 + \epsilon^2 2^{2(n-1)}}. \quad (22)$$

However, if inequality (21) is valid, then from (22)

$$\left| G_f''(j\omega) \right|_{\omega=\frac{1}{\alpha}}^2 \approx \frac{1+\epsilon^2}{K_f^2}. \quad (23)$$

The denominator  $D(s)$  becomes a better approximation to  $K_f^2$  for  $\omega > 1/\alpha$ .

Since the numerator  $N(s)$  for these frequencies swings between 1 and  $1+\epsilon^2$  (see equation (18)), the magnitude-squared function  $|G_f''(j\omega)|^2$  will swing  $1/K_f^2$  and  $(1+\epsilon^2)/K_f^2$ ,  $n$  times. This ripple effect begins at  $\omega = 1/\alpha$  which may be considered the corner frequency of the numerator. The above characteristics are diagrammatically illustrated in figure 11.

Heretofore, just the magnitude function has been studied, because phase information cannot be obtained from magnitude-squared functions. To acquire the phase functions, the roots of the equal-ripple and maximally-flat polynomials must be found. Expressing the magnitude-squared function as

$$\left| G_f(j\omega) \right|^2 = G_f(j\omega) G_f(-j\omega) \quad (24a)$$

or

$$\left| G_f(j\omega) \right|_{-s^2=\omega^2}^2 = G_f(s) G_f(-s), \quad (24b)$$

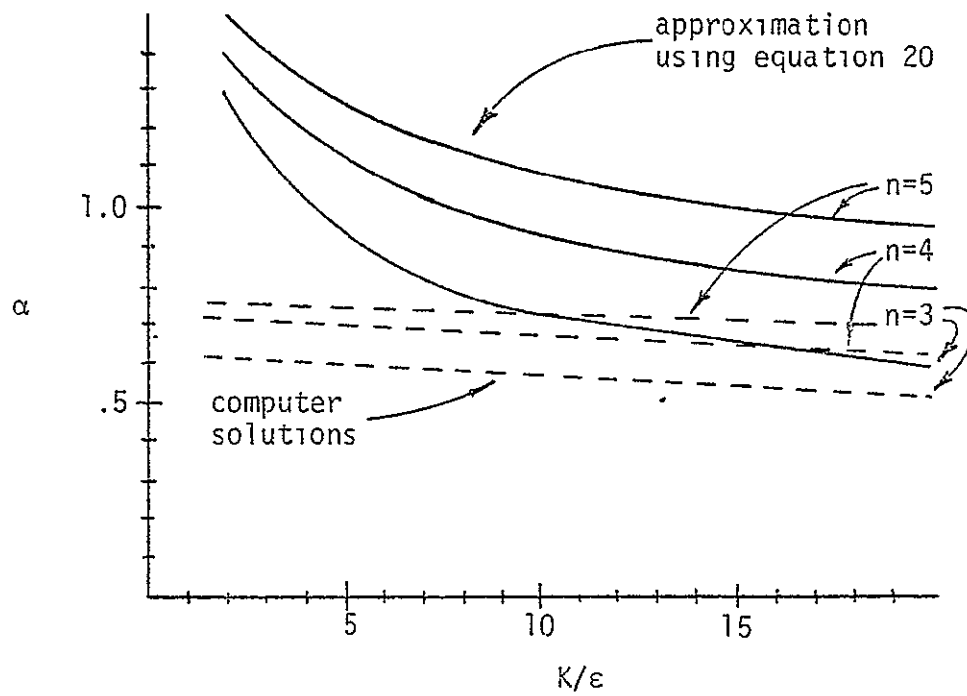


Figure 10

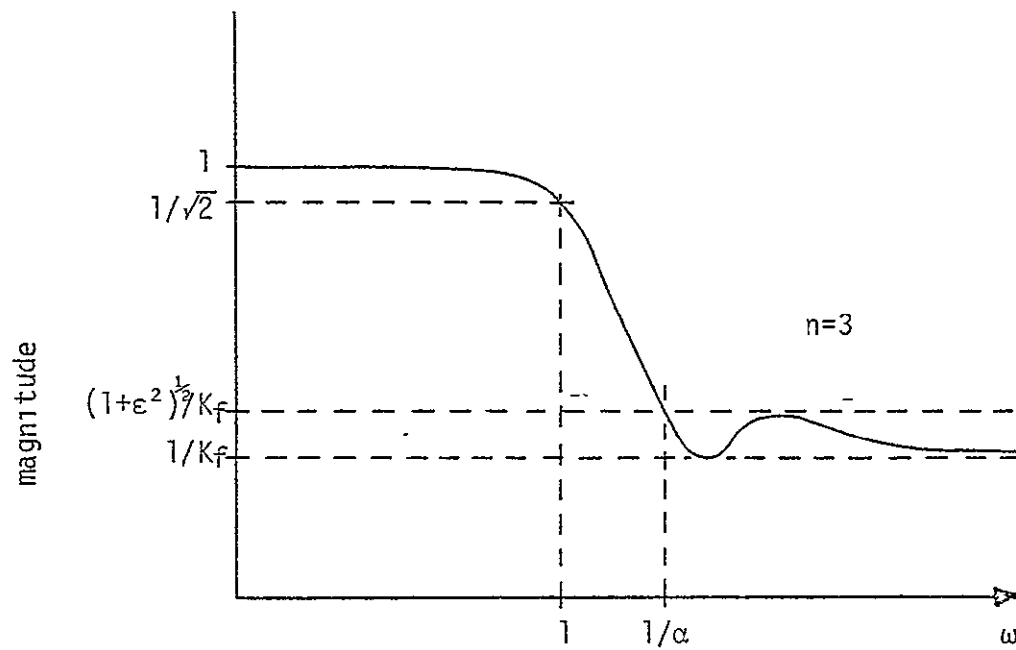


Figure 11

$G_f(s)$  may be found. It should be remembered that although the notation is in terms of the imaginary variable  $j\omega$ , the variable which appears in the function is the real variable  $\omega^2$  (for example see (14)). The desired generalization is made by substituting  $s/j$  for  $\omega$ . The remaining problem is to separate equation (24b) into its two constituents,  $G_f(s)$  and  $G_f(-s)$ . This is done by realizing that  $G_f(s)G_f(-s)$  contains the poles and zeros of  $G_f(j\omega)$  and their mirror images with respect to the  $j\omega$  axis. Therefore, to assure a stable minimal-phase system one assigns all the left half-plane roots to  $G_f(j\omega)$ .

Solving the denominator of equation (14) by substituting  $s/j$  for  $\omega$ ,  $D(s)D(-s)$  becomes

$$D(s)D(-s) = K_f^2 + \epsilon^2 \frac{2^{2(n-1)}}{(-1)^n \alpha^{2n} S^{2n}} = 0. \quad (25a)$$

To simplify the computation, a new variable  $p$  is defined:  $p=1/s$ .

Substituting into (25a),

$$p^{2n} = (-1)^n \frac{\alpha^{2n} K_f^2}{\epsilon^2 2^{2(n-1)}}. \quad (25b)$$

Solving for the  $2n$  roots,

$$p_m = \sqrt[n]{\frac{2^n K_f^2}{\epsilon^2 2^{n-1}}} e^{j \frac{\pi}{2} (1 + \frac{2m-1}{n})}, \quad \text{where } m = 1, 2, 3, \dots, 2n. \quad (25c)$$

The  $n$  left half-plane roots,  $p_k$ , may be found from (25c) to be

$$p_k = \sqrt[n]{\frac{\alpha^n K_f}{\epsilon 2^{n-1}}} [-\sin \frac{\pi}{2n}(2k-1) + j \cos \frac{\pi}{2n}(2k-1)] \quad (25d)$$

, where  $k=1,2,\dots,n$ .

The roots  $p_k$  lie on a circle of radius  $\sqrt[n]{\frac{\alpha^n K_f}{\epsilon 2^{n-1}}}$ . Note that if the normalizing parameter  $\alpha$  is approximated by (20) the circle becomes a unit circle.

The function  $D(s)$  is found by replacing  $p$  with  $1/s$ ;

$$D(s) = C_0 [D(p) \big|_{s=\frac{1}{p}}]. \quad (25e)$$

Since  $D(s)$  is constructed from the roots, and the roots solve all  $p$ -polynomials that are multiples of  $D(p)$ , the constant  $C_0$  is needed. The constant  $C_0$  is found by equating  $D(s)$  with the square root of equation (25a) for any  $\omega$ .

Solving for the roots of the numerator of equation (14) is more difficult.

Substituting  $s/j$  for ,

$$N(s)N(-s) = 1 + \epsilon^2 C_n^2 (j/\alpha s) = 0. \quad (26a)$$

Again utilizing  $p=1/s$  for simplification, equation (26a) can be written

$$1 + \epsilon^2 C_n^2 (\frac{jp}{\alpha}) = 0. \quad (26b)$$

With equation (15a),

$$c_n^2 \left( \frac{jp}{\alpha} \right) = \cos(n \cos^{-1} \frac{jp}{\alpha}) = \pm j \frac{1}{\epsilon}. \quad (26c)$$

In solving for  $p$ , the complex variable  $w$  is introduced,

$$w = u + jv = \cos^{-1} \frac{jp}{\alpha}. \quad (26d)$$

Substituting and equating real and imaginary parts,

$$\begin{aligned} \cos n w &= \cos(nu + jnv) \\ &= \cos(nu) \cosh(nv) - j \sin(nu) \sinh(nv) = \pm j \frac{1}{\epsilon} \end{aligned} \quad (26e)$$

Hence,

$$\cos(nu) \cosh(nv) = 0$$

and

$$\sin(nu) \sinh(nv) = \pm \frac{1}{\epsilon}. \quad (26f)$$

$\cosh(nv)$  cannot be equal to zero for real  $v$ ; therefore,  $\cos(nu) = 0$ . The solutions for  $u$  are

$$u_m = \frac{\pi}{2n} (2m-1), \text{ where } m = 1, 2, \dots, 2n. \quad (26g)$$

For  $u_m$ ,  $\sin(nu_m)$  is equal to one. Solving the remaining equation of (26f),

$$\sinh(nv) = \pm \frac{1}{\epsilon}, \text{ it follows that } v = \frac{1}{n} \sinh^{-1} \frac{1}{\epsilon}. \quad (26h)$$

Rearranging equation (26d)

$$\frac{jp_m}{\alpha} = \cos w = \cos(u_m + jv), \quad \text{where } m=1, 2, \dots, 2n. \quad (26i)$$

The  $n$  left half-plane roots are,

$$p_k = \alpha(-\sin(u_k)\sinh(v) + j\cos(u_k)\cosh(v)), \quad (26j)$$

where  $k = 1, 2, \dots, n$ .

The function  $N(s)$  is found by

$$N(s) = C_1 [N(p) \mid s = \frac{1}{p}].$$

The constant  $C_1$  is needed for the same reasons that  $C_0$  was used in equation (25e).

To realize the benefit of the filter design, one must apply the above results to the noise-reduction systems in the references [1,2,5]. For comparison with [1,2,5] the amplifier gain in figure 2 will be 20dB. The  $G_f'''(j\omega)$  used for this example will be a third-order filter, having a filter gain,  $K_f$ , of -17dB and a ripple of 1.5dB. To obtain the value of  $\epsilon$ ,  $\sqrt{1+\epsilon^2}$  must be made 1.5dB greater than 1, that is

$$20 \log_{10} \sqrt{\frac{1+\epsilon^2}{1}} = 1.5\text{dB}. \quad (27)$$

Here  $\epsilon$  is found to be .6423.

For a filter gain of -17dB,  $K_f$  from equation (17a) is

$$-17\text{dB} = 20 \log_{10} \frac{1}{K_f} \quad (28)$$

where  $K_f$  is found to be 7.079.



In this example, inequality (21) will be considered adequate; then from (20)  $\alpha$  is found to be .7133.

These four parameters ( $n, \alpha, \epsilon, K_f$ ) provide a good idea of what the normalized magnitude function  $|G_f'''(j\omega)|$  will look like; the -3dB bandwidth frequency is at  $\omega=1$ ; the corner frequency, where the ripple effect begins is at  $\omega=1/\alpha$ ; magnitude function will swing between  $1/K_f=.1413$  and  $\sqrt{1+\epsilon^2}/K_f = .1679$  exactly  $n=3$  times. These characteristics are illustrated in figure 11.

Starting with equation (25d) the roots of the  $D(s)$  are found to be

$$p_1 = -.500 + j.866, \quad p_2 = -1.00, \quad p_3 = -.500 - j.866.$$

Multiplying the roots together to form  $D(p)$ ,

$$\begin{aligned} D(p) &= (p+1.00)[(p+.500)^2 + .866^2] \\ &= p^3 + 2.00p^2 + 2.00p + 1. \end{aligned}$$

Using (25e),

$$D(p) \Big|_{s=\frac{1}{p}} = \frac{1}{s^3} + \frac{2.00}{s^2} + \frac{2.00}{s} + 1. \quad (29)$$

Before finding  $D(s)$ ,  $C_0$  of equation (25e) must be determined. As  $\omega \rightarrow \infty$  for real frequencies  $D(p) \Big|_{s=1/p} \rightarrow 1$  (see (29)) but,  $|D(s)| \rightarrow K_f$  (from the square root of (25a)). Therefore,  $C_0$  must be equal to  $K_f$  (see (25e)). It follows that

$$D(s) = 7.079 \left[ \frac{1}{s^3} + \frac{2.00}{s^2} + \frac{2.00}{s} + 1 \right]. \quad (30)$$

Using equation (26j) the roots are found to be

$$p_1 = -.1495 + j.6696, p_2 = -.2990, p_3 = -.1495 - j.6696.$$

Multiplying the roots together to form  $N(p)$ ,

$$\begin{aligned} N(p) &= (p + .2990)[(p + .1495)^2 + .6696^2] \\ &= p^3 + .5980p^2 + .5601p + .1407. \end{aligned}$$

Using (26k)

$$N(p) \Big|_{s=\frac{1}{p}} = \frac{1}{s^3} + \frac{.5980}{s^2} + \frac{.5601}{s} + .1407 \quad (31)$$

As  $C_0$  was evaluated  $C_1$  is found to be  $K_f$ . Thus

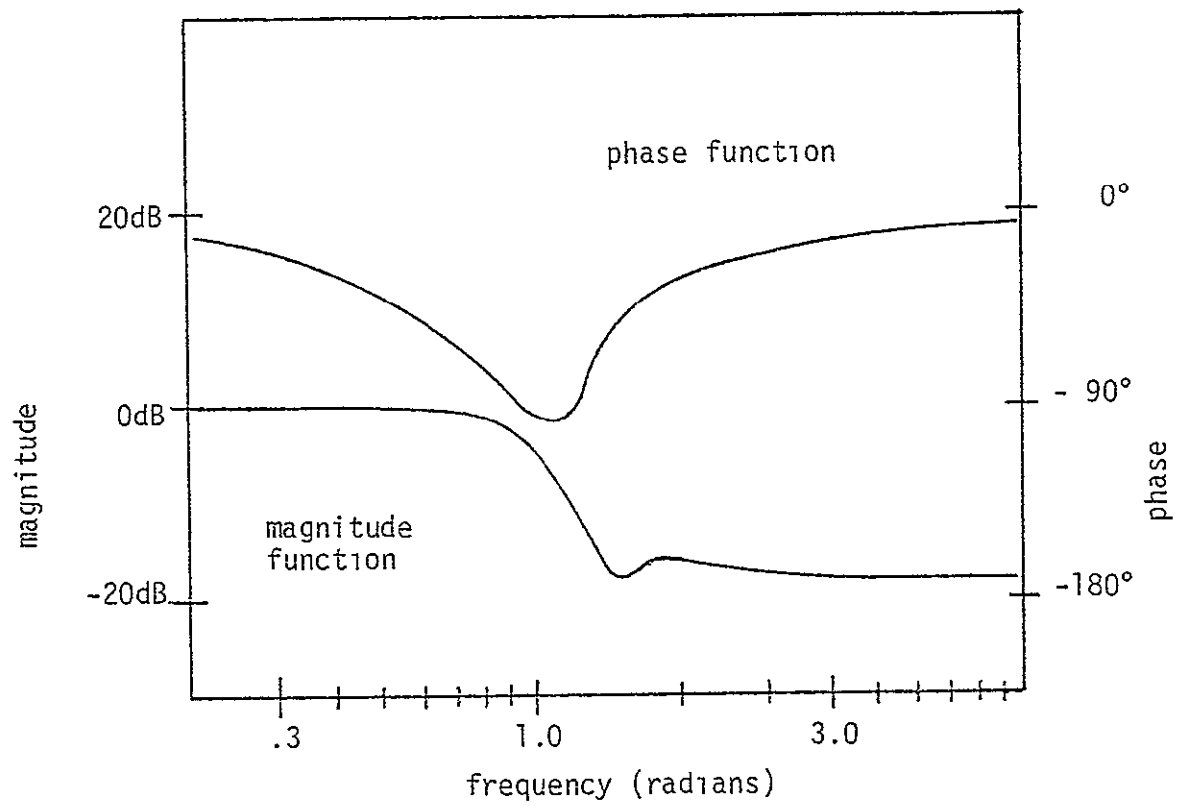
$$N(s) = \frac{7.079}{s^3} + \frac{4.250}{s^2} + \frac{3.981}{s} + 1 \quad (32)$$

Finally,

$$G_f'''(s) = \frac{s^3 + 3.981s^2 + 4.250s + 7.079}{7.079s^3 + 14.16s^2 + 14.16s + 7.079} \quad (33)$$

The magnitude and phase functions of  $G_f'''(j\omega)$  for real frequencies  $\omega$  are shown in figure 12.

$G_f'''(s)$  is now used in the noise-reducing system in figures 3a and 3b (with amplifier gain  $K_a = 20\text{dB}$ ) to reduce the open-loop gain from  $K_a$  to  $K_a/K_f$ . Because the phase of  $G_f'''(s)$  (for large  $n$ ) can shift beyond  $-180$  degrees for some frequencies, the open-loop gain  $|K_a G_t|$  must be kept greater than unity until the open-loop phase returns to above  $-180$  degrees. This is necessary to avoid encirclement of the  $(-1, j0)$  critical



Third-order Butterworth-Chebyshev Filter

Figure 12

point on the Nyquist diagram. Subsequently, the magnitude must become less than unity before the open-loop phase function, which reflects the increasing negative phase of the time delay, returns below -180 degrees, see 13a and 13b.

This magnitude characteristic is not always guaranteed by  $G_m(s)G_s(s)$ . In this case, an additional lag pole will be necessary.

Through graphical manipulation (using figures 3a,3b,12a,12b), the open-loop transfer function characteristics in figure 13a and 13b were constructed. Here, the -3dB frequency  $\omega_{-3dB}$  of  $G_f'''(j\omega)$  is found to be 1.0KHz. The normalized filter function (33) is translated to the actual frequencies by replacing  $s$  in (33) by  $s/2\pi(1000)$ :

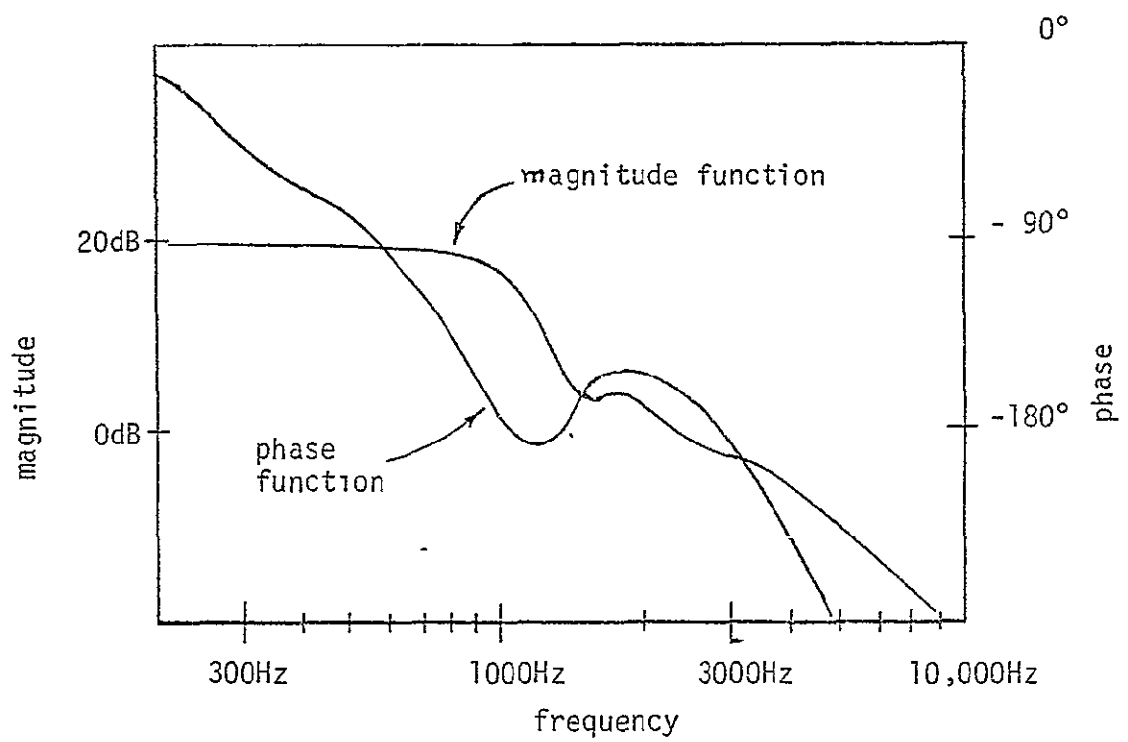
$$G_f'''(s) = \frac{4.030 \times 10^{-12} s^3 + 1.01 \times 10^{-8} s + 6.76 \times 10^{-4} s + 7.079}{2.85 \times 10^{-12} s^3 + 3.59 \times 10^{-7} s^2 + 2.25 \times 10^{-3} s + 7.079} \quad (34)$$

$$\omega_{-3dB} = 2000\pi$$

Also, from figures 13a and 13b, it can be seen that a pole is needed near 2.2KHz to bring the magnitude function below unity. Now the open-loop transfer function in  $s$  for figures 13a and 13b is

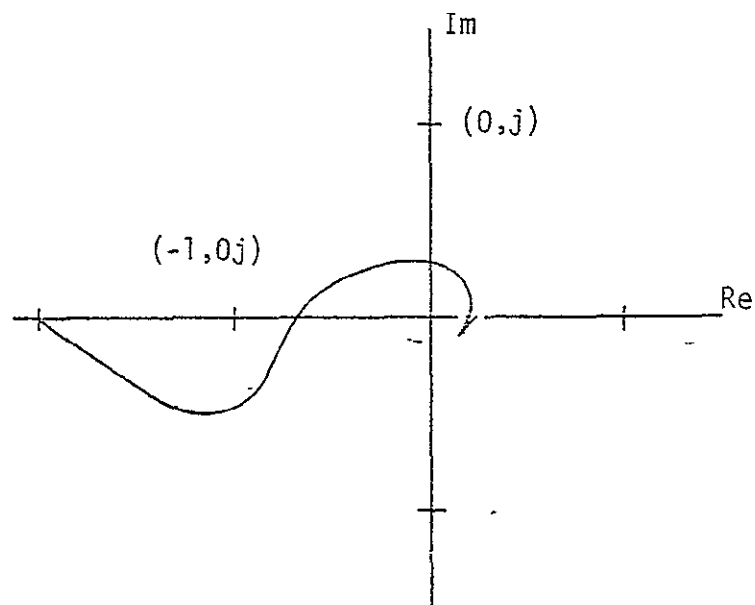
$$|K_a G_t(s)| = 10 \left[ G_f'''(s) \right]_{\omega_{-3dB}=2000\pi} \frac{1}{7.23 \times 10^{-5} s + 1} \frac{1}{(3.98 \times 10^{-5} s + 1)^2} \quad (35)$$

In the above example the bandwidth was extended to 1000Hz with a third-order, maximally-flat, equal-ripple, lag-lead filter.



Olson's system with Butterworth-Chebyshev Compensation

Figure 13a



Nyquist diagram of Figure 13a

Figure 13b

## CONCLUSION

The work reported upon here showed that, by using additional electronic filtering, the bandwidth of an active electronic noise reduction system could be increased by two octaves. This improvement was achieved through the use of the algebraically tractable Butterworth and Chebyshev functions. Also, filter parameters  $(n, \alpha, \epsilon, K_f)$  were chosen such that significant filter characteristics could be determined by inspection. Hence, by employing such an easily designed electronic filter, a significant improvement in active electronic noise reduction systems was realized.

For further improvements in system bandwidth, higher-order filter functions could be tried, in addition to types other than Butterworth and Chebyshev. Moreover, improvements which would reduce time delay in other system components should be investigated.

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